

Problem Set 5

The following exercises are taken from “Dummit & Foote”

1. nn. 2,10 p. 116–117
2. nn. 6,30 p. 130–132

Further exercises

3. Let Q_8 be the group of invertible matrices 2×2 generated by $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ and $\begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$. What is the order of Q_8 ? Show that Q_8 has only one element of order 2. What is the center of Q_8 ? Show that every subgroup of Q_8 is normal in Q_8 .
4. Let G be a finite group and let H be a subgroup of G . Let N be a normal subgroup of G . Show that if H and $[G : N]$ are co-primes, then H is a subgroup of N .
5. Show that S_4 has 3 2-Sylow subgroups isomorphic to D_8 .