

1. Suppose that G is a finite abelian group which can not be written as a nontrivial direct sum $G_1 \oplus G_2$. Show that G is a cyclic group of order p^n for some prime number p .

2. Find the $\text{Hom}(\mathbb{Z}_4 \oplus \mathbb{Z}_6, \mathbb{Z}_5 \oplus \mathbb{Z}_6)$.

3. If $n = p_1^{n_1} \dots p_k^{n_k}$ is a prime factorization of the number n then we define $r(n) = p_1 \dots p_k$. Show that if G is an abelian group of order n then it has a cyclic subgroup of order $r(n)$. Give an example to show that this is not true if G is not abelian.