1. Find the Galois group of the extension $K/\mathbb{Q}$ where K is the splitting field of the following polynomials :

(1) $x^4 - 4x + 2$

(2) $x^3 + 4x + 2$

(3) $x^3 - 10x^2 + 4$

(4) $x^4 + 2$

2. If $\mathbb{Q}_8 = \{\pm 1, \pm i, \pm j, \pm k\} \subseteq \mathbb{H}^*$ is the Quaternion group then show that there exists no Galois extension K/F such that K is the splitting field of a polynomial of degree 4 and $Gal(K/F) \cong \mathbb{Q}_8$.

3. Find a $n \in \mathbb{N}$ such that $\mathbb{Q}(\omega)$, where ω is a primitive nth root of unity, has a subfield which is not a Cyclotomic extension of $\mathbb{Q}$.