1. Suppose that $G < (\mathbb{Z}^2, +)$ is generated by the elements $(1,2), (-1,5)$. Then find the $|\mathbb{Z}^2 : G|$.(*Hint: If we put* $e = (1,0) + G$ *and* $S = \{0, e, 2e, ..., 6e\} \subseteq \mathbb{Z}^2/G$ *then show that* S *has 7 different elements of* $\mathbb{Z}^2/G$ *and* $(0,1) + G \in S$)

2. Show that $(\mathbb{Q}^*, \times)$ is not a free group.

3. Suppose that $G = \mathbb{Z}^r$ for some $r \in \mathbb{N}$. We call the elements of the subset $S = \{s_1, ...s_n\} \subseteq G$ to be linearly independent if

$$n_1 s_1 + ... + n_r s_r = 0 \Rightarrow n_1 = ... = n_r = 0$$

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a) If $\{s_1, ...s_n\}$ form a basis for the group G then show that its elements are linearly independent.

b) If the elements of the subset $S = \{s_1, ...s_n\} \subseteq G$ are linearly independent can we find a subset of G namely $S' \supseteq S$ such that the elements of S' form a basis for G?