1. If G is a finite group and P is a p Sylow subgroup of G, S is a set of p Sylow subgroups of G and we define the action of P on S in the following way

$$a \in P \quad , \quad Q \in S \quad : \quad a.Q = aQa^{-1}$$

show that the only fix element of this action is P. i.e:

$$Q \in S \quad , \quad \forall a \in P \ a.Q = Q \Rightarrow Q = P$$

2. Suppose G is a finite group and $|G| = p^m q^n$ for prime numbers p, q

a) If P is a p Sylow subgroup of G then the number of p Sylow subgroups of G is q^k for some $0 \leq k \leq n$

b) If

$$p \nmid q^k - 1 \qquad k = 1, ..., n$$

then p Sylow subgroup P of G is normal in G.

3. If G is a finite group, P is a p Sylow subgroup of G and H is a subgroup of G so that $N(P) \subseteq H$ then $N(H) = H$. [*Hint: If $a \in N(H)$ then $aPa^{-1} \subseteq H$ is a p Sylow subgroup of H and so can be written as hPh^{-1} for some $h \in H$*]

4. Let $R = \mathbb{Z}[\sqrt{-6}]$. Prove that 2 and $\sqrt{-6}$ are irreducible in R. Show that R is not UFD. Give an explicit ideal in R that is not principal.

5. In the ring of the Gaussian integers find $GCD\{11 + 7i, 18 - i\}$.