1. If $R = \mathbb{Z}[\omega]$ where $\omega = e^{\frac{2\pi i}{3}}$ then

a) show that $1 - \omega$ is prime. (*Hint*: find $N(1 - \omega)$)

b) show that $N(a + b\omega) = a^2 - ab + b^2$ and then find all unit elements of R

c) if $q \equiv 2 \mod 3$ is a prime number then it is prime in R

d) find a prime factorization for 7

2. If $R = \mathbb{Z}[\sqrt{2}]$ then show that the group of unit elements of R is infinite.(*Hint*: first show that $1 + \sqrt{2}$ is a unit)

3. If R is a ring with a multiplicative function $N : R \to \mathbb{N}$ with the property that x is unit iff $N(x) = 1$ then every element can be written as a product of irreducible elements. we don't need the uniqueness.

4. Show that the equation

$$ax + by = c$$

has solutions in $\mathbb{Z}$ if and only if GCD$\{a, b\}$ divides c.

Does the equation

$$12x + 7y = 26$$

have integer solutions? If yes, then determine the complete set of solutions.

5. Find the inverse, if exists, of $\bar{X}$ in the quotient ring

$$\mathbb{R}[X]/(X^2 + X + 1)$$