

1. Show that the set $\{1, \sqrt{2}, \sqrt{3}\}$ is made by elements of \mathbb{R} that are linearly independent over \mathbb{Q} .

2. Show that

$$\mathbb{Q}(\sqrt{2}, \sqrt{3}) = \mathbb{Q}(\sqrt{2} + \sqrt{3})$$

3. If F is a field and $\text{Char}F \neq 2$ and E/F is an algebraic field extension of degree 2, then there exists an element $\alpha \in E$ such that $\alpha^2 = a \in F$ and $E = F(\alpha)$.

4. Suppose that E/F is a field extension. If for some $x \in E$ we have $F(x) = F(x^2)$ then show that x is algebraic over F .

5. Exercise 4, p. 352 of the textbook

6. Exercise 5, p. 352 of the textbook.