1. If $F \subseteq E$ are fields and for some $x \in E$ the $|F(x) : F|$ is odd then

$$F(x) = F(x^2)$$

2. If F, K, L are fields and

$$|L : F| = n \quad , \quad |K : F| = m$$

then show that

$$|KL : F| \leq mn$$

.

3. In the above problem show that the equality holds if $(m, n) = 1$.

4. If p is a prime number and $\omega = e^{\frac{2\pi i}{p}}$ then show that

$$|\mathbb{Q}(\sqrt[p]{2}, \omega) : \mathbb{Q}| = p(p-1)$$

Hint: you may need to use the fact that $1 + x + ... + x^{p-1}$ *is the minimal polynomial of* ω

5. Show that $|\mathbb{Q}(\sqrt[4]{2}, \sqrt{3}) : \mathbb{Q}| = 8$.