1. Find the splitting field E of the following polynomials over $\mathbb{Q}$ and find the degree $|E : \mathbb{Q}|$.

(a) $x^3 + 1$
(b) $x^4 + 1$

2. Do problem 1 if we replace $\mathbb{Q}$ by $\mathbb{Z}_2, \mathbb{Z}_3$.

3. Find the splitting field of the polynomial $x^4 - 7$ over the fields $\mathbb{Q}, \mathbb{Z}_5, \mathbb{Z}_{11}$.

4. If $q \equiv 3 \mod 4$ is a prime number then show that

$$\mathbb{F}_{q^2} = \{a + bi : a, b \in \mathbb{Z}_q\}$$

where the addition and multiplication is defined by the following rules :

$$(a_1 + b_1 i) + (a_2 + b_2 i) = (a_1 + a_2) + (b_1 + b_2)i$$

$$(a_1 + b_1 i)(a_2 + b_2 i) = (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1)i$$

5. Determine the minimal polynomial over $\mathbb{Q}$ for the element $1 + \sqrt[3]{6}$

6. Determine the degree over $\mathbb{Q}$ of $1 + \sqrt{7} + \sqrt[3]{4}$