1. (a) If $n \in \mathbb{N}$ then $\cos \frac{2\pi}{n}, \sin \frac{2\pi}{n}$ are algebraic over $\mathbb{Q}$

(b) Show that $\mathbb{Q}(\cos \frac{2\pi}{n})/\mathbb{Q}$ is a Galois extension. Compute its Galois group

(c) Is $\mathbb{Q}(\sin \frac{2\pi}{n})/\mathbb{Q}$ a Galois extension ?

(*Hint: It maybe helpful to note that* $e^{\frac{2\pi i}{n}} = \cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}$. *For b it is enough to show that* $Gal(\mathbb{Q}(\omega)/\mathbb{Q}(\cos \frac{2\pi}{n}) \trianglelefteq Gal(\mathbb{Q}(\omega)/\mathbb{Q}\ ))$

2. If $F = \mathbb{Q}(\sqrt[4]{2}, i)$ then show that $F/\mathbb{Q}$ is Galois and find its Galois group.

3. Find the Galois group of the splitting field of the following polynomials over $\mathbb{Q}$:

(a) $x^3 - 3x + 1$

(b) $x^3 + x + 1$