Thesis Abstract: Abhishek Banerjee

(Johns Hopkins University (PhD expected: March 2009))

Title: Nearby Cycles, Archimedean Complex and Periodicity in Cyclic Homology

This thesis consists of four parts;

(1) In the first part, we compare the monodromy on the nearby cycles complex ψ^{**} to the periodicity operator in cyclic homology of a sheaf of differential operators. The nearby cycles complex is quasi-isomorphic to the complex A^{**} of Steenbrink that determines the limiting mixed Hodge structure on the nearby fibre for an algebraic degeneration over a disc. By stacking copies of the nearby cycles complex ψ^{**}, we construct a triple complex $\mathcal{BC}^{***}$ whose "rows" are quasi-isomorphic to the cyclic homology complex of the sheaf of differential operators and show that the periodicity operator on the cyclic homology of the latter is identical to the monodromy on $\mathcal{BC}^{***}$ induced from the ψ^{**}.

(2) In the second part, we implement the arithmetic analogue of the same construction, using objects $K^{i,j,k}$ which were constructed by Consani to correspond to direct summands of the E_0-terms of the spectral sequence associated to the Picard-Lefschetz filtration on Steenbrink's complex A^{**}. In this case, we show that the periodicity operator in cyclic cohomology of the ring of differential operators corresponds to the monodromy on Consani's complex.

Further, we assemble the summands $K^{i,j,k}$ to form bicomplexes φ^{**} and B^{**}, which correspond to complexes ψ^{**} and A^{**} in part (1) respectively, together with a morphism $\mu : \varphi^{**} \longrightarrow B^{**}$. Both complexes φ^{**} and B^{**} carry monodromy operators N and we show that $N \circ \mu - \mu \circ N$ is homotopic to zero.

(3) In the third part, we define the algebraic, topological and relative K-theories of the sheaf of differential operators on a locally Stein manifold and demonstrate a long exact sequence connecting them that lifts the periodicity sequence in the cyclic (hyper)homology of the sheaf of differential operators. We also show that similar constructions may be made for formal schemes.

(4) In the final part, we construct an "enriched archimedean complex" on the modular tower that extends Consani's complex $K^{i,j,k}$ from part (2). We show that this archimedean complex carries an action of the Connes-Moscovici's Hopf Algebra $\mathcal{H}_1$ of codimension 1 foliations as well as a bimodule structure over a slight variant $\mathcal{A}_T$ of Connes-Moscovici's modular Hecke algebra $\mathcal{A}$. We also show that the action of the Hopf algebra $\mathcal{H}_1$ is "flat".

Finally, we show that the enriched archimedean complex carries Rankin Cohen brackets of all orders.