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Kholles

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1 First dish

Exercise 1. Show that the set whose elements are the functions $f_{\varepsilon,b}\begin{cases}\mathbb{R}^2 \to & \mathbb{R}^2 \\ (x,y) \mapsto & (\varepsilon x + b, y)\end{cases}$ with $\varepsilon \in \{\pm 1\}, b \in \mathbb{R}$ equipped with the law $\circ$ is in fact a non-commutative (non abelian) group. Give a geometrical interpretation of this group.

Exercise 2. Show that the set whose elements are the functions $f_{\varepsilon,a}\begin{cases}\mathbb{R}^2 \to & \mathbb{R}^2 \\ (x,y) \mapsto & (x + a, \varepsilon y)\end{cases}$ with $\varepsilon \in \{\pm 1\}, a \in \mathbb{R}$ equipped with the law $\circ$ is in fact a commutative (abelian) group. Give a geometrical interpretation of this group.

Exercise 3. We define two new operations on $\mathbb{R}^2$, $\oplus : \begin{cases}\mathbb{R}^2 \times \mathbb{R}^2 \to & \mathbb{R}^2 \\ ((a_1,b_1),(a_2,b_2)) \mapsto & (a_1 + a_2, b_1 + b_2)\end{cases}$ and $\otimes : \begin{cases}\mathbb{R}^2 \times \mathbb{R}^2 \to & \mathbb{R}^2 \\ ((a_1,b_1),(a_2,b_2)) \mapsto & (a_1 a_2, a_2 b_1 + a_1 b_2)\end{cases}$

1. Describe the set $(\mathbb{R}^2)^\times$ of the elements of $\mathbb{R}^2$ having an inverse for $\otimes$. (And so $((\mathbb{R}^2)^\times, \otimes)$is a group).
2. Show that the map $e : \begin{cases}\mathbb{R}^2 \to & (\mathbb{R}^2)^\times \\ (a,b) \mapsto & (\exp(a), b\exp(a))\end{cases}$ is an homomorphism of groups.
3. Describe $\ker(e)$.

2 Second Dish

Exercise 4. Draw the Cayley graph of the group of symmetries of an equilateral triangle (this group is called the 3^{rd} dihedral group, denoted D_3 and can be identified with $(S_3, \circ)$ where S_3 denotes the set of of bijections from $[\![1,3]\!]$ to $[\![1,3]\!]$ and $\circ$ the composition of functions). *Hint :* you can check the rotation of one third of a turn (whose center is the center of the triangle) and the axial symmetry along the height going through one fixed summit are enough to generate this group.

Exercise 5. Draw the Cayley graph of $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$. Compare with the Caley graph of S_2

Exercise 6. Draw the Cayley graph of the group of symmetries of a square (this group is called the dihedral 4^{th} dihedral group and denoted D_4). *Hint :* you can check the rotation of one forth of a turn (whose center is the center of the square) and the axial symmetry along a fixed diagonal of the square are enough to generate this group.