8. Sol. ① \( f(3) = a \), so \( f(x) \) is defined at \( x = 3 \).

\[
\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{3+2x-x^2}{x-3} = \lim_{x \to 3} \frac{-(x+1)(x-3)}{x-3} \quad \text{(factor the numerator)}
\]

Thus, \( \lim_{x \to 3} f(x) \) exists.

③ \( a = f(3) = \lim_{x \to 3} f(x) = -4 \).

\[ \Rightarrow a = -4. \]

26. Sol. (a)

When \( c = 0 \)

\[ f(x) = \begin{cases} \frac{1}{x}, & x \neq 1 \\ 2x, & x < 1 \end{cases} \]

As we can see from the graph, there is a jump at \( x = 1 \), so \( f(x) \) is discontinuous at \( x = 1 \).

(b) It suffices to choose \( c \) such that \( f(x) \) is continuous at \( x = 1 \).

i) \( f(1) = 1 \)

ii) \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{1}{x} = 1 \).  \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2x + c = 2 + c \).

To guarantee that \( \lim f(x) \) exists, we must let

\[ \lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) \], that is to say, \( 2 + c = 1 \Rightarrow c = -1 \).

iii) Then \( f(1) = \lim_{x \to 1} f(x) \). So we must choose \( c = -1 \).
46. \[
\lim_{x \to 0} \frac{5 - \sqrt{25 + x^2}}{2x^2}
\]
\[
= \lim_{x \to 0} \frac{5 - \sqrt{25 + x^2}}{2x^2} \cdot \frac{5 + \sqrt{25 + x^2}}{5 + \sqrt{25 + x^2}}
\] (Rationalize the numerator)
\[
= \lim_{x \to 0} \frac{-x^2}{2x^2(5 + \sqrt{25 + x^2})}
\]
\[
= \lim_{x \to 0} \frac{-1}{2(5 + \sqrt{25 + x^2})} = \frac{-1}{2(5 + 5)} = -\frac{1}{20}.
\]

8. \[
\lim_{x \to 0} \frac{3 - x^2}{2 - 2x^2} = \lim_{x \to 0} \frac{-x^2}{-2x^2} = \lim_{x \to 0} \frac{1}{2} = \frac{1}{2}.
\]

**Proof:** Since \( f \) & \( g \) are continuous at \( x = c \),

we have

\[
\lim_{x \to c} f(x) = f(c)
\]
\[
\lim_{x \to c} g(x) = g(c)
\]

Apply the limit law, we get:

\[
\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{f(c)}{g(c)} \quad \text{provided that} \quad \lim_{x \to c} g(x) = g(c) \neq 0.
\]

So \( \frac{f}{g} \) is continuous at \( x = c \).