Section 4.3

30. Differentiate
\[ f(x) = \frac{r + x}{rs^2} - rsx + (r + s)x - rs \]
with respect to \( x \). Assume \( r \) and \( s \) are nonzero constants.

First, we can apply the Sum and Difference Rule, which allows us to write:
\[
\frac{df}{dx} = \frac{d}{dx} \left[ \frac{r + x}{rs^2} \right] - \frac{d}{dx}[rsx] + \frac{d}{dx}(r + s)x - \frac{d}{dx}rs.
\]

\( 1/(rs^2) \), \( rs \), and \( (r + s) \) are all constants, so we can apply the Scalar Multiple Rule to the first term. The last term is a constant, so its derivative is zero.

\[
\frac{df}{dx} = \frac{1}{rs^2} \frac{d}{dx}[r + x] - rs \frac{d}{dx}[x] + (r + s) \frac{d}{dx}[x].
\]

Then the derivative of \( x \) is 1, as is the derivative of \( r + x \).

\[
\frac{df}{dx} = \frac{1}{rs^2} - rs + (r + s).
\]

We can simplify this in a variety of ways, which I have omitted.

Section 4.4

44. Assume that \( f(x) \) and \( g(x) \) are differentiable at \( x \). Find an expression for the derivative of \( y \).

\[ y = [-2f(x) - 3g(x)]g(x) + \frac{2g(x)}{3}. \]

The first term is a case of the Product Rule, and the second, of the Constant Multiple Rule.

\[
y' = \left( \frac{d}{dx}[-2f(x) - 3g(x)] \right) g(x) + [-2f(x) - 3g(x)] \frac{d}{dx}g(x) + \frac{2g'(x)}{3}.
\]

The term in the large parenthesis can be solved by the Sum and Constant Multiple Rules.

\[
y' = [-2f'(x) - 3g'(x)]g(x) + [-2f(x) - 3g(x)]g'(x) + \frac{2g'(x)}{3}
\]

\[
= -2f'(x)g(x) - 2f(x)g'(x) - 6g'(x)g(x) + \frac{2g'(x)}{3}
\]

There are other solutions, such as beginning by distributing the original function.

Section 4.5

22. Differentiate:

\[ h(x) = \sqrt[3]{1 + 2x}. \]

This is a case of the Chain Rule, with \( h(x) = f(g(x)) \) for \( f(x) = \sqrt[3]{x} \), \( g(x) = 1 + 2x \). The derivative of \( \sqrt[3]{x} \), which is also \( x^{1/3} \), is a Power Rule case:

\[ f'(x) = \left( \frac{1}{3} \right) x^{-2/3}. \]
And the derivative of \( g(x) \) is 2. So, the Chain Rule gives us:

\[
h'(x) = f'(g(x))g'(x)
\]

\[
= \left( \frac{1}{3} \right) (1 + 2x)^{-2/3}(2)
\]

\[
= \left( \frac{2}{3} \right) (1 + 2x)^{-2/3}.
\]

38. Find:

\[
\frac{d}{dx} f[g(x) + 1] = f'[g(x) + 1]g'(x).
\]

We apply the Chain Rule, remembering that the derivative of \( g(x) + 1 \) is equal to that of \( g(x) \).

\[
\frac{d}{dx} f[g(x) + 1] = f'[g(x) + 1][g(x) + 1]' = f'[g(x) + 1]g'(x).
\]

Section 4.6

20. Assume that the radius \( r \) and the area \( A = \pi r^2 \) of a circle are differentiable functions of an independent variable \( t \). Express \( dA/dt \) in terms of \( dr/dt \).

Since \( A \) and \( r \) are functions of \( t \) and are equal, we can take their derivatives to be equal as well:

\[
\frac{dA}{dt} = \frac{dr}{dt} \pi r^2.
\]

The right-hand side is a case of the Chain Rule, with squaring as the outer function and \( r(t) \) as the inner function. The derivative of \( t^2 \) is \( 2t \), so we find:

\[
\frac{dA}{dt} = (\pi)(2r) \frac{dr}{dt} = 2\pi r \frac{dr}{dt}.
\]