1. Do not open this exam until you are told to begin.

2. This exam has 6 pages including this cover. There are 5 questions.

3. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.

4. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.

5. Please turn off all cell phones and pagers and remove all headphones.

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Instructor: José Manuel Gómez

Johns Hopkins University
1. (10 Points) Compute the following limits.

(a) (2 Points) \( \lim_{x \to 2^+} \frac{x}{(2 - x)^{11}} \)

(b) (3 Points) \( \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \)

(c) (2 Points) \( \lim_{x \to \infty} \frac{x^2 - x}{3x^3 - x + 1} \)

(d) (3 Points) \( \lim_{x \to 0} \frac{1 - \cos(2x)}{3x} \)
2. (10 Points) Let

\[ f(x) = \sqrt{4x^2 - 16}, \]
\[ g(x) = \frac{x}{x^2 - 9}. \]

(a) (2 Points) Is the expression \((f \circ g)(0)\) defined? If so what is it?

(b) (4 Points) Find the domain of the function \(g \circ f\).

(c) (2 Points) Determine if the following sequence is convergent or divergent.

\[ a_n = \frac{n^2 + 1}{n^3}. \]

(d) (2 Points) Find the limit of the sequence \(b_n\) defined below, in case it exits.

\[ b_n = 1 + \frac{3^n + 1}{5^n}. \]
3. (10 Points) The function \( f \) is defined by

\[
f(x) = \begin{cases} 
\sin(2\pi x) + 4c & \text{if } x < 2, \\
8 & \text{if } x = 2, \\
x^2 + c^2 & \text{if } 2 < x.
\end{cases}
\]

(a) (2 Points) Find \( \lim_{x \to 2^-} f(x) \).

(b) (2 Points) Find \( \lim_{x \to 2^+} f(x) \).

(c) (6 Points) Find the value of \( c \) so that the function \( f \) is continuous at \( x = 2 \). Please explain your answer.
4. (10 Points) A scientist wants to study a certain kind of radioactive isotope present in a fossil. Let $P(t)$ denote the amount (in grams) of this isotope present in the fossil $t$-years after its discovery. Using a detailed study the scientist was able to derive the equation

$$P(t) = 1200 \left( \frac{1}{4} \right)^{t/120}.$$

(a) (2 Points) What was the initial quantity in grams of the radioactive isotope present in the fossil when it was discovered?

(b) (4 Points) What is the half life of this particular kind of isotope?

(c) (4 Points) The amount of the radioactive isotope present today in the fossil is 100 grams. How long ago was the fossil discovered?
5. (10 Points) Consider the function $h(x)$ defined as follows

$$h(x) = \begin{cases} 
  x^2 \sin\left(\frac{\pi}{x}\right), & \text{if } x \neq 0; \\
  -2, & \text{if } x = 0.
\end{cases}$$

(a) (6 Points) Is the function $h$ continuous at $x = 0$?

(b) (4 Points) Is the function $w(x) = \frac{x}{x + 1}$ one to one? Please explain your answer.