1) Sketch of graph of $f$
2) c.s.i. $f''(x) \text{ DNE} \rightarrow \text{Crit pts}$
3) c.s.i. $f''(c) = 0$\[\text{loc min} \quad \text{loc max}\]

First deriv test for loc extrema at endpoints $f$ is able on $[a,b]$ (1-sided at endpoints xy pts deriv at $x=a$ & $x=b$).
If $f'(a) > 0$ then $f$ has loc min at $x = a$ < loc max < loc min
If $f'(b) < 0$ then $f$ has loc max at $x = b$ < loc min

Picture.

Second deriv test for loc extrema $f$ is able on an open interval $c$ in $[a,b]$.\[f''(x) < 0 \rightarrow \text{loc min}\]

Picture.

Find extrema of $f(x) = xe^{-x}$ on $[0,10]$
Solu: $f'(x) = e^{-x} - xe^{-x} = (1-x)e^{-x}$
At endpoints: $f'(0) = 1 > 0 \rightarrow \text{loc min at } x = 0$
$f'(10) = -9e^{-10} < 0 \rightarrow \text{loc min at } x = 10$

Crt pts.: solve $0 = (1-x)e^{-x} \rightarrow x = 1$
$f''(x) = -e^{-x} - (x-1)e^{-x} = (1-x)e^{-x}$
f''(1) = -e^{-1} < 0 \rightarrow \text{loc max at } x = 1$

Note: $f(0) = 0, f(1) = \frac{1}{e}, f(10) = 10e^{-10}$

Fact: If $f$ is twice able $f$ has inf pt at $c$.
Then $f''(c) = 0$
Note: the cond $f''(c) = 0$ only gives candidates for inf pts.

Ex: $f(x) = x^5, f''(x) = 20x^3$ inf pt at $0$
Ex: $f(x) = x^4, f''(x) = 12x^2$ poss on both sides of $x = 0$ so 0 is not an inf pt.

Ex: $f(x) = xe^{-x}$ on $\mathbb{R}$. Find inf pt.
From before: $f''(x) = (x-2)e^{-x}$
Solve $0 = (x-2)e^{-x} \Rightarrow x = 2$
$f''(x) > 0$ if $x > 2$ \Rightarrow only inf pt.