Solutions to \( ax^2 + bx + c = 0 \) are:
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
- could be real
- complex numbers

13 Elementary Functions

A function \((f:A \to B)\) is a rule assigning to each elt \(x\) in a set \(A\) a unique elt \(y = f(x)\) (the image of \(x\)) in a set \(B\). Write: \(f: A \to B, \ x \mapsto f(x)\)

A called domain of \(f\)

\(B\) (codomain or target of \(f\)

\(f(A) = \{y \in B | y = f(x) \text{ some } x \in A\}\) range of \(f\)

"Set of vals taken by \(f\)"

\(f(A) = \{y \in B | y = f(x) \text{ some } x \in A\}\)

\(f: (0,\infty) \to \mathbb{R}, \ x \mapsto \sqrt{x}\) range is \((0,\infty)\)

For \(A, B\) subsets of \(\mathbb{R}\), the graph of \(f\) is:
\[(x, y) \in \mathbb{R}^2 | y = f(x)\]

Vertical line test: a curve \(C \subset \mathbb{R}^2\) is the graph of a

\(f\) if each vertical line intersects \(C\) at most once.

**Def:** \(f: A \to B\) is **even** if \(f(-x) = f(x) \forall x \in A\) (eg: \(x^n\))

**Odd** if \(f(-x) = -f(x) \forall x \in A\) (eg: \(\sin x\))

**Def:** Let \(g: A \to B \land f: C \to D\). The composite function \(f \circ g\) is defined by:
\[(f \circ g)(x) = f(g(x)) \forall x \in A \text{ s.t. } g(x) \in C\]

For all such \(x\)

\[(f \circ g)(x) = f(g(x)) \forall x \in A \text{ s.t. } g(x) \in C\]

**Def:** A polynomial is a function of form:
\[f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0\]

\(a_0, a_1, \ldots, a_n \in \mathbb{R}\)

**If** \(f \neq 0\), biggest \(d\) s.t. \(a_d \neq 0\) is degree of \(f\). 

\(a_0\) is leading coefficient 

Max dom of \(f = \mathbb{R}\)

A rational fun is fun of form:
\[f(x) = \frac{p(x)}{q(x)}\]

\(p, q\) poly's \& \(q \neq 0\).

Max dom of \(f\) is \([x \in \mathbb{R} | q(x) \neq 0]\)

**Def:** An exponential fun \(y = a^x\) \(a \neq 0\), \(a \neq 1\) of form
\[f(x) = a^x, \ \text{ } a \text{ const}\]

Max dom \(= \mathbb{R}\)

"Best" base: \(a = e\), \(e \approx 2.718\), \(f(x) = e^x\)

**Def:** \(f: A \to B\) is **one-to-one** (1-1)

- \(f(x) \neq f(y)\) if \(x \neq y\)

- \(f(x) = x^2, \text{ not } 1-1\) on \(\mathbb{R}\) by \(f(1) = f(-1) = 1\)

- \(f(x) = x\) is 1-1 on \((0,\infty)\)

**Def:** \(f: A \to B\) is **onto** (surjective)

\[f: A \to B \text{ with } B = \text{ range of } f(A)\]

**The inverse fun** \(f^{-1}: f(A) \to A\) def by:
\[f^{-1}(y) = x \iff y = f(x) \land y \in f(A)\]

\(f(x) = x^2\) on \((0,\infty)\). Range \([0,\infty)\), 

\[f^{-1}(x) = \sqrt{x}\]

**Def:** \(a > 0, a \neq 1\). The logarithm of base \(a\) \(\log_a(x)\) is the inverse of \(f(x) = a^x\), def on \((0,\infty)\)

4 Graphing

- **Basic eq's**: