More good ex's in the book!

§5.10 Antiderivatives

Q: Given $f'$, can we recover $f$?

I.e. can solve $\frac{df}{dx} = g(x)$, given $g$?

Eq Newton: \[ F = ma \]

Derive $v$ from this?

Def An antiderivative for $f$ on interval $I$ is a $F$ s.t. $F'(x) = f(x)$ \quad $\forall x \in I$.

Eq $f(x) = \cos x$. Antideriv is $F(x) = \sin x$.
Another is $F(x) = \sin x + 2$.

Eq $f(x) = \tan x$. Antideriv is $F(x) = -\ln|\cos x| + C$.

Recall from §5.1: $g$ cont on $(a,b]$ & dble on $(a,b)$ & $g'(x) = 0 \quad \forall x \in (a,b)$ $\Rightarrow g$ cont on $[a,b]$.

Cor Supp $F$ & $G$ antideriv of $f$ on interval $I$. Then $f$ cont on $I$. $F(x) = G(x) + C \quad \forall x \in I$.

pf \[ \frac{(F - G)'(x)}{G(x) - F(x)} = \frac{F'(x) - G'(x)}{G(x) - F(x)} = 0 \]

conti & dble $\Rightarrow G(x) = F(x) + C$ some cont $C$.

So often write general antider as (particular antider) + $C$.

Eq General antider of

$e^x$ is $xe^x + C$.
$e^x$ is $\frac{1}{2}xe^x + C$.
$\sin x$ is $\cos x + C$.
$\cos x$ is $-\sin x + C$.
$\tan x$ is $\ln|\cos x| + C$.
$\sec^2 x$ is $\tan x + C$.
Ex $x^2 + 1$ is $\int x^2 - 1$ from $(0,0)$ to $(a,a)$.

Not a single interval!
Strictly speaking, should have different $c$ on each interval. Usually ignore this in the notation.

Rem

1) No gen procedure/final for finding antiderivs

2) Some haven't have "easy to write" antiderivs

\[ f(x) = e^{-x^2} \]

3) Can specify the const $C$ if given "initial cond"

\[ \int f(x) \quad dx \]

\[ f(0) = 2 \]

Gen soln $f(x) = \frac{2}{3}x^{3/2} + C$.

$2 = f(1) = \frac{2}{3} + C \Rightarrow C = \frac{4}{3}$.

\[ f(x) = \frac{2}{3}x^{3/2} + \frac{4}{3} \]

Every differentiable fun is a contideral funa in opp direction.

For particular antideriv

\[ \begin{align*}
\int x^n \; dx & = \frac{1}{n+1}x^{n+1} + C \\
\int \frac{1}{x} \; dx & = \ln|x| + C \\
\int e^x \; dx & = e^x + C \\
\int \sin x \; dx & = -\cos x + C \\
\int \cos x \; dx & = \sin x + C \\
\int \tan x \; dx & = -\ln|\cos x| + C \\
\int \sec x \; dx & = \ln|\sec x + \tan x| + C \\
\end{align*} \]

Ch 6: Integration

§6.1 The Definite Integral

Fundamental problem 1: given $f$ on $[a,b]$, the area under $f$...