§2.1 Exponential growth & decay

Eq of population growth:

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pop size</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>400</td>
</tr>
</tbody>
</table>

\[ N(t) = 50 \cdot 2^t, \quad t = 0, 1, 2, \ldots \]

Gen case: \( f(x) = N_0 a^x \)

- \( N_0 = f(0) \) initial value
- \( a \) = factor value must be \( \neq 1 \) each unit change of \( x \) (growth/decay const)

\[
\begin{array}{cc}
N_0 & a > 1 \\
\text{exp growth} & \text{exp decay} \\
N_0 & 0 < a < 1 \\
\end{array}
\]

For large "populations" (or whatever), often

\( x \) - can be discrete or continuous

\( a = 1 \) boring! (usually don't allow)

\( a > 1 \):

\( a = 50, \quad a_n = 2 \cdot a_{n-1} \)

For small values, use a continuous variable to interpolate betw discrete vals.

For eqs \( f: \mathbb{R} \to \mathbb{R} \quad a > 0 \quad \& \quad a \neq 1 \)

\( x \mapsto N_0 a^x \)

\( \text{dom} = \mathbb{R}, \quad \text{range} = (0, \infty) \)

Note on shapes:

\[
\begin{array}{cc}
5^x & 2^x \\
\end{array}
\]

§2.2 Sequences

Informally, a sequence is \( (a_0, a_1, a_2, \ldots) \quad a_n \in \mathbb{R} \)

\( (50, 100, 200, 400, \ldots) \)

Formally:

\( N_0 = \{0, 1, 2, \ldots\} \)

Def: A sequence is a fun \( f: \mathbb{Z}_{\geq 0} \to \mathbb{R} \)

\( n \mapsto f(n) \)

For such \( f \), get tuple \( (f(0), f(1), f(2), \ldots) \)

as before (look sometimes drops parentheses)

Often represent seq as \( (a_n) \)

Don't always have to start at \( 0 \),

eg \( (a_n) \) where \( a_n = \frac{1}{n}, \quad n = 1, 2, \ldots \) (a_0 not def!)

Can be explicit about starting index:

\( (b_n)_{n=0} = \left( \frac{1}{n^2} \right)_{n=1} \)

Sometimes def seq recursively, i.e. via a fixed \( a_0 \) fixed

\( a_n = g(a_{n-1}) \quad n = 1, 2, \ldots \)

\( (a_n)_{n=0} = 50 \quad a_n = 2 \cdot a_{n-1} \)

\( \Rightarrow a_4 = 2 \cdot 50 = 100 \quad a_3 = 2 \cdot 100 = 200 \quad \ldots \)

"implies" seq is \( (50, 100, 200, 400, \ldots) \)

\( a_n = 50 \cdot 2^n, \quad n = 0, 1, \ldots \)