To get slope of tan line, let \( h \to 0 \)

Definition: The derivative of a function \( f \) at \( c \) is

\[
f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}
\]

(provided limit exists)

\( f \) is differentiable at \( c \) when this limit exists at a particular \( c \) and differentiable when able at all \( c \) in its dom.

\[
f'(c) = \frac{f(c+h) - f(c)}{h}
\]
called difference quotient

Change var: \( x = c+h \)

\[
f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x-c}
\]

Alt notation: for \( y = f(x) \), write

\[
y' = \frac{dy}{dx} = f'(x) = \frac{df}{dx} = \frac{df}{dx}
\]

\[
f'(c) = \frac{df}{dx} \bigg|_{x=c}
\]

\[\text{Eqn of tan line: if } f'(c) \text{ exists, then tan line to graph of } f \text{ at } x=c \text{ is line through } (c, f(c)) \]

\[\text{w/ slope } f'(c).
\]

\[\text{Pt-slope form: } y - f(c) = f'(c)(x - c)
\]

\[\text{Eqn } f(x) = x^2
\]

\[\text{tan line at } (1,1): f'(1) = 2; y - 1 = 2(x-1)
\]

\[\text{tan line at } (-2,4): f'(-2) = 4; y - 4 = 4(x+2)
\]

\[\text{Example } f(x) = ax + b
\]

Slope had better be a ! (at x)

\[\text{Eqn } f(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{x+1}, & x \geq 0 \end{cases}
\]

\[\text{Rem. A straight line is its own tan line at every pt.}
\]

\[\text{Ex. 2 Properties of the deriv}
\]

- If \( s(t) \) is position as function of \( t \), \( \frac{ds}{dt} \) is velocity
- If \( v(t) \) is any quantity as function of \( t \), then \( \frac{dv}{dt} \) is rate of change

\[\text{Eqn } f(x) = 2 - |x-1| = \begin{cases} 2x - 1, & x < 1 \\ -x + 3, & x \geq 1 \end{cases}
\]

\[\Rightarrow f'(1) \text{ DNE.}
\]

\( f \) is cts at \( x=1 \), so we see cts \( \neq \) able

But converse is true

Then If \( f \) able at \( x=c \), then \( f \) cts at \( x=c \).

\[\text{First show } \lim_{x \to c} f(x) = f(c)
\]

\[\text{Well, } \lim_{x \to c} f(x) = \lim_{x \to c} [f(x) - f(c) + f(c)]
\]

\[= \lim_{x \to c} \left[ \frac{f(x) - f(c)}{x-c} (x-c) + f(c) \right]
\]

\[= \lim_{x \to c} f(x) - f(c) + f(c)
\]

\[= f'(c) \cdot 0 + f(c) = f(c)
\]

\[\text{Lin laws OK be all lims along the way exist.}
\]

\[\text{Eqn } f(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{x+1}, & x \geq 0 \end{cases}
\]

\[\text{Discs at 0, so not defl at}
\]