110.107 CALCULUS 2
FALL 2016
MIDTERM 2

Name: ________________________________

Recitation section:

__ 1. Tuesday 4:30 (S. Harrop)
__ 2. Tuesday 3:00 (S. Harrop)
__ 3. Thursday 3:00 (C. Kauffman)
__ 4. Thursday 1:30 (C. Kauffman)

Work quickly and carefully, and write your solutions clearly. Please show your work; partial credit will be given generously.

Statement of ethics
I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ____________________________ Date: ________________

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<th>Problem</th>
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Problem 1. [10 points] Let $f(x,y) := e^x \ln y$, defined on the set of $(x,y) \in \mathbb{R}^2$ such that $y > 0$. Compute the linearization of $f$ at the point $(1,2)$. 
Problem 2. [10 points] Let \( f(x, y) := 3x^2y + 5xy - y^3 \). Evaluate the directional derivative of \( f \) at \((-1,1)\) with respect to the unit vector pointing in the direction of greatest increase of \( f \) at this point.
Problem 3. [10 points] Find the critical points of the function $f(x, y) := 3x^2 - 6xy + y^2$ on $\mathbb{R}^2$ and determine whether each is a local maximum, a local minimum, or a saddle point.
Problem 4. [10 points] Use the method of Lagrange multipliers to find the maximum value attained by the function $f(x, y) := 2x + 4y^2$ on the unit circle $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$. 
Problem 5. [10 points] Find the general solution to the first order linear system

\[
\frac{dx}{dt} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} x(t),
\]

and find all equilibria solutions and evaluate their stability.
Problem 6. [10 points]

(a) An urn contains six blue and four black balls. How many ways are there to choose (without replacement) three balls such that all are of the same color?

(b) Paula, Cindy, Gloria, Anne, and Jenny have dinner at a round table. In how many ways can they sit around the table if Cindy wants to sit to the left of Paula?