Calc II Bio
HW 1 Solutions
By Caroline VanBlargan

7.4 # 10, 12, 18, 26, 40

10. The integral is improper b/c it has an unbounded integral

\[ \int_{-\infty}^{\infty} x^3 e^{-x^4} dx = \int_{-\infty}^{0} x^3 e^{-x^4} dx + \int_{0}^{\infty} x^3 e^{-x^4} dx \]

\[ \text{since } x^3 e^{-x^4} \text{ is an odd function} \]

We were given that the integral converges, so we can immediately conclude that the terms cancel out so

\[ \int_{-\infty}^{\infty} x^3 e^{-x^4} dx = 0 \]

12. \[ \int_{\frac{1}{\sqrt{\ln x}}}^{e} \frac{e^x}{x \ln x} \, dx \] is improper b/c the integrand becomes unbounded as \( x \to 1 \)

\[ \int_{\frac{1}{\sqrt{\ln x}}}^{e} \frac{e^x}{x \ln x} \, dx = \int_{0}^{1} \frac{1}{u^{1/2}} \, du = \lim_{a \to 0} \int_{a}^{1} u^{1/2} \, du = \lim_{a \to 0} \frac{2u^{3/2}}{3} \bigg|_{a}^{1} \]

\[ = \lim_{a \to 0} 2 - 2a^{3/2} = 2 - 0 = 2 \]
18. \( \int_{1}^{\infty} \frac{1}{x^{3/2}} \, dx \) diverges

Notice for \( x \geq 1 \), \( x^{1/2} \leq x^{3/5} \), so \( x^{1/2} \leq \frac{1}{x^{3/5}} \), and from example 2 (pg 353) \( \int_{1}^{\infty} \frac{1}{x^{5/6}} \, dx \) diverges.

So \( \int_{1}^{\infty} \frac{1}{x^{3/2}} \, dx \) diverges from the comparison test.

26. \( \int_{1}^{\infty} \frac{e^x}{x \ln x} \, dx = \int_{1}^{\infty} \frac{1}{u} \, du \)

\[ = \lim_{a \to 0} \ln u \bigg|_{a}^{1} \]

\[ = 0 - \infty \rightarrow \text{diverges} \]

40. Since \( \sqrt{1 + x^6} \geq \sqrt{x^6} \),

\( \frac{1}{\sqrt{1 + x^6}} \leq \frac{1}{\sqrt{x^6}} = \frac{1}{|x^3|} = \frac{1}{x^3} \) (for \( x \geq 1 \)).

And since \( \int_{1}^{\infty} \frac{1}{x^3} \, dx \) is convergent (pg 354),

\( \int_{1}^{\infty} \frac{1}{\sqrt{1 + x^6}} \, dx \) converges.