11.2 Problem 26

Transform the second-order differential equation
\[
\frac{d^2x}{dt^2} = -\frac{1}{2}x
\]
into a system of first-order differential equations.

To do this, we introduce another function \( v(t) = x'(t) \), so that \( \frac{dx}{dt} = v \). We also note by using our second-order differential equation that \( \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\frac{1}{2}x \). Then, our system of differential equations is:

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} x \\ v \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}
\end{align*}
\]

and this is a system of first-order differential equations, as desired.

12.1 Problem 14

A committee of 3 people must be chosen from a group of 10. The committee consists of a president, a vice president, and a treasurer. How many committees can be selected?

There are 10 possible choices for the president, then 9 for the vice president and 8 for the treasurer, so that there are \( 10 \cdot 9 \cdot 8 = 720 \) possible committees to form. We can view this as a “permutation”-type problem because the three positions on the committee are distinguishable.

12.1 Problem 30

Twelve children are divided up into three groups, of five, four, and three children, respectively. In how many ways can this be done if the order within each group is not important?

\[
\frac{12!}{5!4!3!},
\]

because there are \( 12! \) ways to order the children, and if we put the first 5 children of our ordering in the first group, the next four in the second group, and the last three in the last group, we will obtain all possible groupings but we will over-count: we can rearrange the children in the first group in \( 5! \) ways without affecting the groupings, the children in the second group in \( 4! \) ways without affecting the groupings, and the children in the last group in \( 3! \) ways without affecting the groupings.

Another way to see this is by choosing 5 children to be in the first group and then 4 of the remaining 7 to be in the second group, leaving the last 3 for the third. There are \( \binom{12}{5} \binom{7}{4} \) to choose 4 out of 7, so that we arrive at \( \binom{12}{5} \binom{7}{4} = \frac{12!}{5!7!} \frac{7!}{4!3!} = \frac{12!}{5!4!3!} \) ways to put the children into groups as desired.

12.1 Problem 46

In how many ways can two aces and three kings be drawn from a standard deck of cards if cards are drawn without replacement?

There are \( \binom{4}{2} = 6 \) ways to choose 2 aces from the deck, and \( \binom{4}{3} = 4 \) ways to choose 3 kings from the deck (drawing without replacement both times), so there are \( 6 \cdot 4 = 24 \) ways to draw two aces and three kings from a standard deck of cards without replacement.
12.2 Problem 12

Assume that \( \Omega = \{1, 2, 3, 4, 5\} \) and that \( P(1) = 0.1, P(2) = 0.2, \) and \( P(3) = P(4) = 0.05. \) Let \( A = \{1, 3, 5\} \) and \( B = \{2, 3, 4\} \). Find \( P(A \cup B). \)

We notice that \( A \cup B = \{1, 3, 5\} \cup \{1, 2, 4\} = \{1, 2, 3, 4, 5\} = \Omega \) and recall that \( P(\Omega) = 1. \) Therefore \( P(A \cup B) = 1. \)