Mathematic 104, Fall 2010: Assignment #3

Due: **Wednesday, October 20th**

Instructions: Please ensure that your answers are legible. Also make sure that all steps are shown – even for problems consisting of a numerical answer. Bonus problems cover advanced material and, while good practice, are *not* required and will *not* be graded.

Problem #1. Excercise 2.6 of Lecture 2 of Trefethen-Bau.

Problem #2. Excercise 6.3 of Lecture 6 of Trefethen-Bau.

Problem #3. Let E be the space in $\mathbb{R}^3$ spanned by

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

a) Find the matrix $P \in \mathbb{R}^{3\times 3}$ corresponding to orthogonal projection onto E.
b) Find a unit vector $q \in \mathbb{R}^3$ so that P is the complementary projector to P_q. Recall $P_q = qq^\top$ is the projector onto the space spanned by q.

Problem #4. Let $A \in \mathbb{C}^{m\times n}$ matrix and $B \in \mathbb{C}^{n\times k}$ matrix. Show that a necessary and sufficient condition for $R(AB) = R(A)$ is that $N(A) + R(B) = \mathbb{C}^n$.

Problem #5. Let $v = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \in \mathbb{R}^2$.

a) Find the orthogonal projector $P_v \in \mathbb{R}^{2\times 2}$.
b) Find the orthogonal matrix $U \in \mathbb{R}^{2\times 2}$ so that $P_v = UXU^\top$ where X is a diagonal matrix with non-negative entries on the diagonal. (Hint: Think geometrically).

Bonus Problem. Let $E, F \subset \mathbb{C}^n$ be two vector spaces. Show that

$$\dim(E + F) = \dim E + \dim F - \dim(E \cap F).$$

(Hint: Let $v_1, \ldots, v_k$ be a basis E and $w_1, \ldots, w_l$ be a basis of F. Consider the $n \times (k + l)$ matrix $X = \begin{bmatrix} v_1 | & \cdots & | v_k | w_1 | & \cdots & | w_l \end{bmatrix}$).