Mathematic 104, Fall 2010: Assignment #4

Due: Wednesday, November 3rd

Instructions: Please ensure that your answers are legible. Also make sure that all steps are shown – even for problems consisting of a numerical answer. Bonus problems cover advanced material and, while good practice, are not required and will not be graded.

Problem #1. Consider the following following matrix:

\[ A = \begin{bmatrix} 3 & 1 \\ -4 & 0 \\ 0 & -1 \end{bmatrix} \]

a) Compute (by hand) a reduced QR factorization of \( A \).

b) Solve in the least squares sense the following two problems and determine which solution is closer to being a true solution:

\[ Ax = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad Ax = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

Problem #2. Given the following 3 points in the plane \( \mathbb{R}^2 \) determine the equation for the line that best fits these points in the sense of least squares. (We didn’t discuss this in class, please read Example 11.2 in Trefethen-Bau).

\[ p_1 = (2, 1), \ p_2 = (0, -1) \quad \text{and} \quad p_3 = (5, 2) \]


Problem #4. Excercise 7.2 of Lecture 7 of Trefethen-Bau.

Problem #5. Excercise 3.3 of Lecture 3 of Trefethen-Bau.

Bonus Problem. Excercise 10.4 of Lecture 10 of Trefethen-Bau.