Mathematic 104, Fall 2010: Assignment #7

Due: **Wednesday, December 3rd**

Instructions: Please ensure that your answers are legible. Also make sure that all steps are shown – even for problems consisting of a numerical answer. Bonus problems cover advanced material and, while good practice, are *not* required and will *not* be graded.

Problem #1. Excercise 2.5 of Lecture 2 of Trefethen-Bau. (In part a) you may use the Schur factorization, in part c) it is helpful to think about the factorization $(I - S)(I + S) = I - S^2$).

Problem #2. Excercise 3.2 of Lecture 3 of Trefethen-Bau. (Here $||A||$ means the induced matrix norm on A from $|| \cdot ||$ on $\mathbb{C}^m$).

Problem #3. Consider the matrix:

$$A = \begin{bmatrix} 1 & 2 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 & 1 \end{bmatrix}$$

Find the singular values of A. You do not need to compute the full SVD. (Hint: There is a right way to do this and a wrong way).

Problem #4. Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} r\cos\theta & -r\sin\theta \\ r\sin\theta & r\cos\theta \end{bmatrix} \in \mathbb{R}^{2\times 2}.$$

Here $r > 0$.

Problem #5. Let

$$A = \begin{bmatrix} 13 & 9 \\ -16 & 37 \end{bmatrix} \text{ and } B = \begin{bmatrix} -35 & 45 \\ 20 & 10 \end{bmatrix}.$$

a) Compute the Schur factorization of A and of B. (Hint: Use the proof of Theorem 24.9 of Trefethen-Bau and the fact that the characteristic polynomials of A and B are easy to factor to find these factorizations).
b) Determine, for both A and B, whether the matrix is diagonalizable. If it is not explain why not and if it is diagonalize it.

Bonus Problem. We say a matrix $A \in \mathbb{C}^{m\times m}$ is *normal* if $A^*A = AA^*$. Show using the Schur factorization that if A is normal then A is unitarily diagonalizable. That is $A = Q\Lambda Q^*$ for Λ diagonal and Q unitary.