Math 104 **Fall 2009**

Homework 4

Due Wednesday, November 4, 2009

1. *Very simple linear regression.* Consider the three points

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -2 \end{bmatrix}.$$

   What is the equation of the line that best fits those three points, in the sense of least squares?

2. *Simple linear regression.* You have pairs of real-valued data (x_i, y_i), $i = 1, \ldots, n$ (for instance, x_i may be the square footage of a home and y_i its sale price). What is the equation of the regression line

$$y = a + bx$$

   in terms of the x_i's and y_i's? The regression line is the line that best fits the points in a least squares sense, i.e. that minimizes $\sum_{i=1}^{n}(y_i - a - bx_i)^2$.

3. *Projections*

   (a) Check that $A(A^*A)^{-1}A^*$ defines an orthogonal projector (assume that A^*A is invertible). What does this project onto?

   (b) Check that $A^*(AA^*)^{-1}A$ also defines an orthogonal projector (assume that AA^* is invertible). What does this project onto?

4. Exercise 4.4, Chapter 4 of Trefethen-Bau (justify your answer).

5. Exercise 5.4, Chapter 5 of Trefethen-Bau.