Problem (6(a)). There exists a point $z \in (a, b)$ such that $|f(z)| = \max_{x \in [a, b]} \{|f(x)|\}$, and clearly $f'(z) = 0$. Fix a point $x \in (a, b)$, WLOG suppose $x \leq z$, then by MVT, there exists some $y \in [a, x]$ such that $f'(y) = f(x)/(x - a)$. Applying MVT to $f'(y) = f'(y) - f'(z)$, we see that $|f'(y)| \leq M_2|y - z|$, so $|f(x)| \leq M_2|x - a||y - z|$. Clearly if $|y - z| \leq |x - b|$ then we are done. Now if $|y - z| > |x - b|$, then $|z - b| < |x - a|$, also there exists some $m \in [z, b]$ for which we have

$$|f'(m)| = \left| \frac{f(z)}{z - b} \right| > \left| \frac{f(x)}{x - a} \right| = |f'(y)|.$$  

Now by IVT for $f'$ we see there is some $\hat{y} \in [z, m]$ such that $|f'(\hat{y})| = |f'(\hat{y})|$, then we have

$$|f'(y)| = |f'(\hat{y})| = \left| \int_z^{\hat{y}} f''(t)dt \right| \leq M_2|\hat{y} - z| \leq M_2|x - b|,$$

therefore

$$|f(x)| \leq M_2|x - a||x - b|.$$