Class 3: 9/6/17 Chapter 1.

Function types

(I) Polynomials $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_0, \ldots, a_n \in \mathbb{R}$ are constants, n = degree.

If domain is unspecified, then all of $\mathbb{R}$.

(II) Rational functions $f(x) = \frac{p(x)}{q(x)}$

where $p(x), q(x)$ are polynomials

Here $q(x) \neq 0$. This helps determine domain.

Note: For $f(x) = \frac{x-1}{x^2-4x+3} = \frac{x-1}{(x-1)(x-3)}$, domain still cannot include $x=1$.

(III) Power functions $f(x) = x^r$, $r \in \mathbb{R}$.

- These are examples of polynomial fncs (called monomials) when $r \in \mathbb{N}$. Otherwise NOT a polynomial.

- Domain: depends on the value of r:
  - $r \in \mathbb{N}$
  - $r > 0$, but $r \notin \mathbb{N}$
  - $r < 0$

(IV) Exponential functions: $f(x) = a^x$, $a > 0, a \neq 1$

- variable is in the exponent. Different from power fncs.
- Why the restrictions on a??
- Domain? Range?

(V) Logarithmic functions: $g(x) = \log_a x$

- domain? $x > 0$. Why?

$$y = a^x \iff x = \log_a y$$

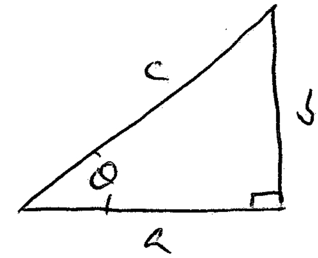
what power to raise a to to get y.

Note: There is a special value for a, $e \cong 2.71828\ldots$
Why this special will be clear later, but the corresponding notation is

$g(x) = \log_e x = \ln x$ natural logarithm.

VI Trigonometric functions:

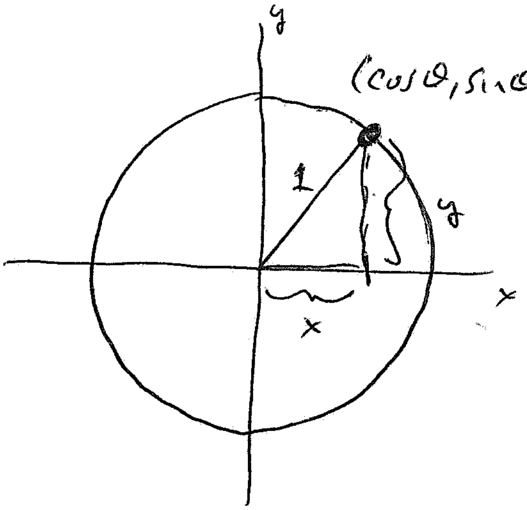
For any right triangle,



one can define functions relating the angle θ ~~sides~~ of this triangle to the sides

$$\sin\theta = \frac{\text{opp side}}{\text{hyp}} = \frac{b}{c} \qquad \cos\theta = \frac{\text{adj side}}{\text{hyp}} = \frac{a}{c}.$$

For the unit circle in the plane, any pt on the circle forms a right triangle wrt its coordinates

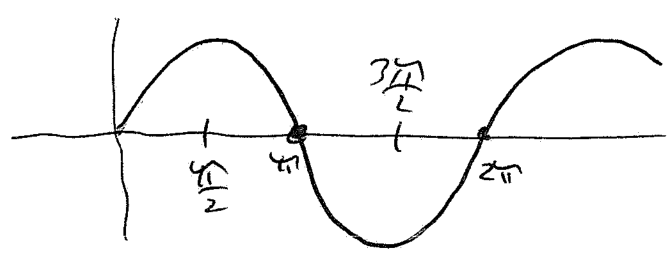


$$x = \frac{x}{1} = \frac{\text{adj side}}{\text{hyp}} = \cos\theta$$

$$y = \frac{y}{1} = \frac{\text{opp side}}{\text{hyp}} = \sin\theta$$

One can simply define a function $f: \mathbb{R} \to \mathbb{R}$

$f(x) = \sin x$



Def A function $f(x)$ is <u>periodic</u> if there is an $a > 0$ so that $f(x+a) = f(x)$ for every x in the domain of f.

The smallest such a is called the <u>period</u> of f.

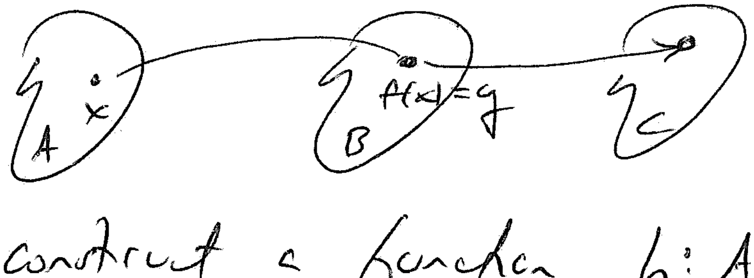
- $f(x) = \sin rx$, for $r \in \mathbb{R}$, $r \neq 0$, is $\frac{2\pi}{r}$ periodic.
- Same for all of the others. But be careful of the domains!

VII Composite fncs: Many fncs are not elementary but are combinations of elementary functions.

Let $f: A \to B$ and $g: B \to C$

Then we can write

$$A \xrightarrow{f} B \xrightarrow{g} C \qquad g(y) = g(f(x))$$



and construct a function $h: A \to C$,

$$h(x) = g(f(x)).$$

What is the domain of h??

V

Here, it is all of A. In general, it will be all pts in the domain of f where $f(x)$ is in the domain of g.

ex. Let $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 1 - x^2$, and

$g: [0, \infty) \to \mathbb{R}$, $g(x) = \sqrt{x}$.

Q: What is the domain of $h(x) = g(f(x))$?

A: Domain of h is: all pts in the domain of the inside function whose function values are in the domain of the outside func.

Here all inputs to $f(x)$ whose outputs are not negative: ~~these~~ all $x \in \mathbb{R}$, where $1 - x^2 \geq 0$. or all $x \in [-1, 1]$.

$h(x) = \cancel{f} \circ g(f(x)) = \sqrt{f(x)} = \sqrt{1 - x^2}$.

Q: What is ~~fog~~ $f(g(x))$? its domain?