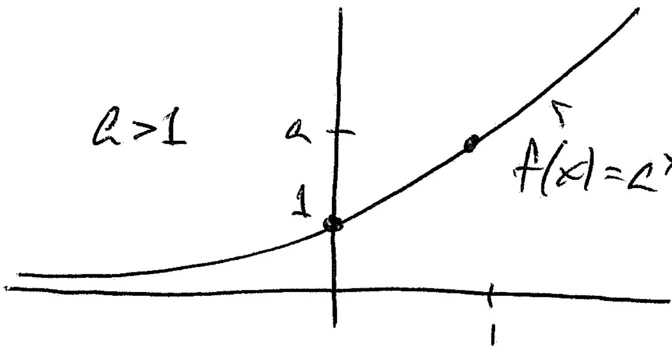
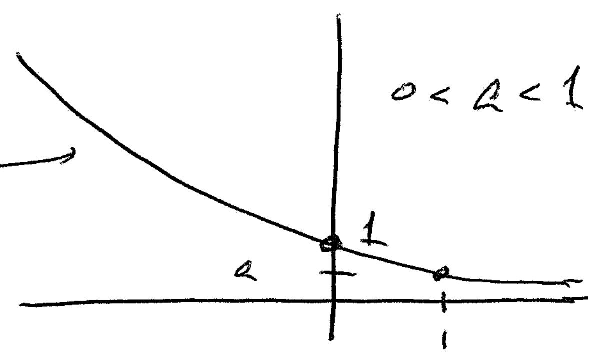
Class 4: 09/08/17

Section 2.1 deals with the structure of exponential functions $f(x) = a^x$, $a > 0$, $a \neq 1$.



exponential growth

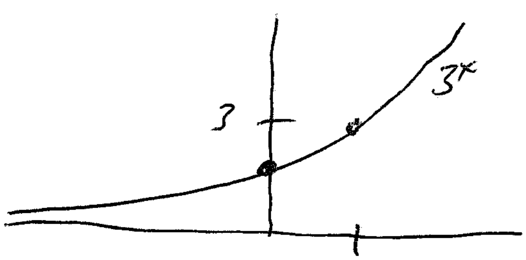


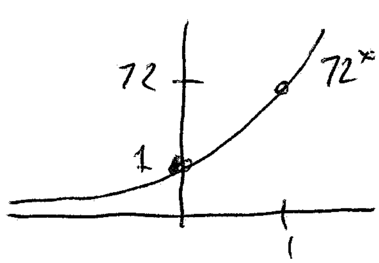
exponential decay

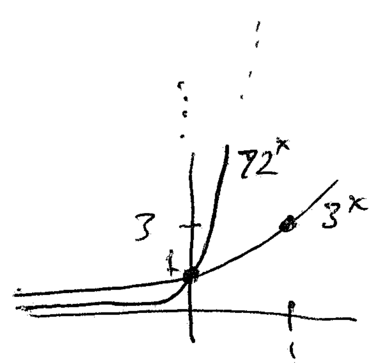
In both cases, $f : \mathbb{R} \to \mathbb{R}$ where domain is all $x \in \mathbb{R}$ and range is only $\mathbb{R}_+ = \{ y \in \mathbb{R} \mid y > 0 \}$.

(There are no solutions to $f(x) = a^x = 0$ or $f(x) = a^x < 0$!)

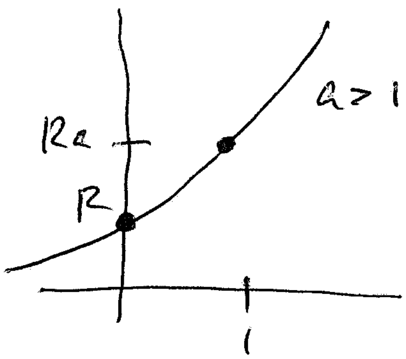
Note about shapes:



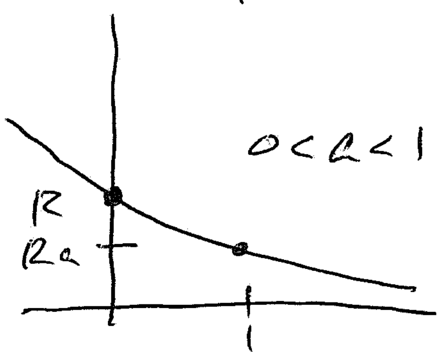




Used to model radio carbon dating, population dynamics as $f(x) = Ra^x$, where

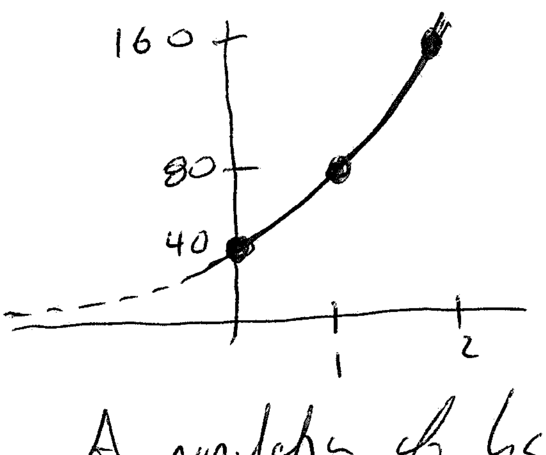


- a is the growth constant $(a>1)$
  decay constant $(1<a<0)$
- R is the initial amount
- x is time,
  - continuous if domain an interval, or
  - discrete if takes only ~~integer~~ natural # values representing regular moments in time.



ex. Section 1.2 #58

$f(t) = 40 \cdot 2^t$, $t \geq 0$. Here $t \in [0, \infty)$

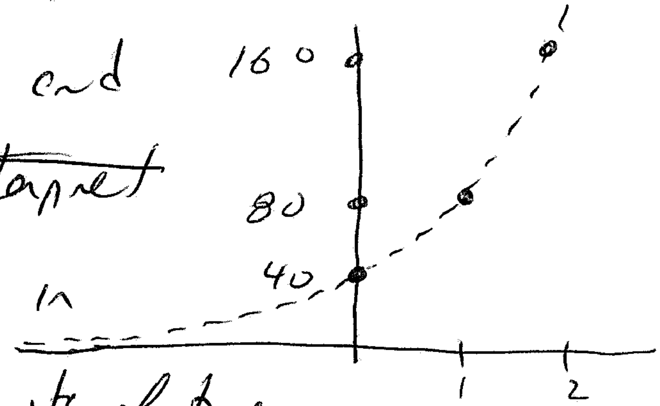


But we can alter the problem for a particular application:

ex. Altered 1.2#58.

A population of bacteria doubles in time every 20 minutes (ex on pg 62). Then $f(t) = 40 \cdot 2^t$, $t \in \mathbb{N}$

Then the model only makes sense at these integer values, and we need to interpret the input values in terms of the units of time.



After an hour ($t=3$), there are $f(3) = 40 \cdot 2^3 = 320$ bacteria.

Note: Sometimes we can interpolate between defined values, but usually only when populations are large.

We call this latter case a discrete model, and can write $f: \mathbb{N} \to \mathbb{R}$, $f(n) = 40 \cdot 2^n$ to accentuate this discreteness.

Given any such function on $\mathbb{N}$, we create an infinite list of numbers:

$$\{f(0), f(1), f(2), \dots\}$$

~~called a sequence~~.

A <u>sequence</u> is any infinite list of numbers

Many notational ways to denote:

- $\{1, 2, 4, 8, \pi, e-1, \dots\}$
- Via a variable and a subscript: (functionally).

$$\{a_n\} = \{a_0, a_1, a_2, \dots\} = \{a_n\}_{n=0}^{\infty}$$

May or may not start at $n=0$

ex. $a_n = \frac{1}{n}$, so $\{\frac{1}{n}\}$ can only start at $n=1$.

ex. Can also say, for $f(x) = \frac{1}{x}$, $a_n = f(n)$

or $f: \mathbb{N} \to \mathbb{R}$, $f(n) = \frac{1}{n} = a_n$

$$\{a_n\} = \{1, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \dots, \tfrac{1}{n}, \dots\}$$

ex. $b_n = (-1)^n (2^n + 1)$, $\{b_n\} = \{2, -3, 5, -9, 17, \dots\}$

ex. $\{N_i\} = 40 \cdot 2^i$, (back to population dynamics)

- Recursively let $f: \mathbb{R} \to \mathbb{R}$ $f(x) = ax$, $a > 0$, $a \neq 1$.

define a sequence: $N_0 = 10$, $N_{i+1} = f(N_i)$.

Can only construct the next term once the previous is known: To find N_4, one first needs N_3 which relies on N_2, ...

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let $a = 3$:

$$N_0 = 10, \quad N_1 = f(N_0) = 3N_0 = 3 \cdot 10 = 30$$
$$N_2 = f(N_1) = 3N_1 = 3 \cdot 30 = 90 = 3^2 \cdot N_0$$
$$N_3 = f(N_2) = 3 \cdot 90 = 270 = 3^3 \cdot N_0$$
$$N_4 = f(N_4) = 3 \cdot 270 = 810 = 3^4 N_0$$

Here, $N_i = N_0 \cdot 3^i$

~~Multiplicit~~ A recursively defined sequence where each term is a constant times the previous term is ~~exp~~ discrete exponential growth!

- ex: Write out the first few terms of $\{b_n\}$ where $b_{n+1} = \frac{1}{4} b_n + \frac{3}{4}$, $b_0 = 2$.