

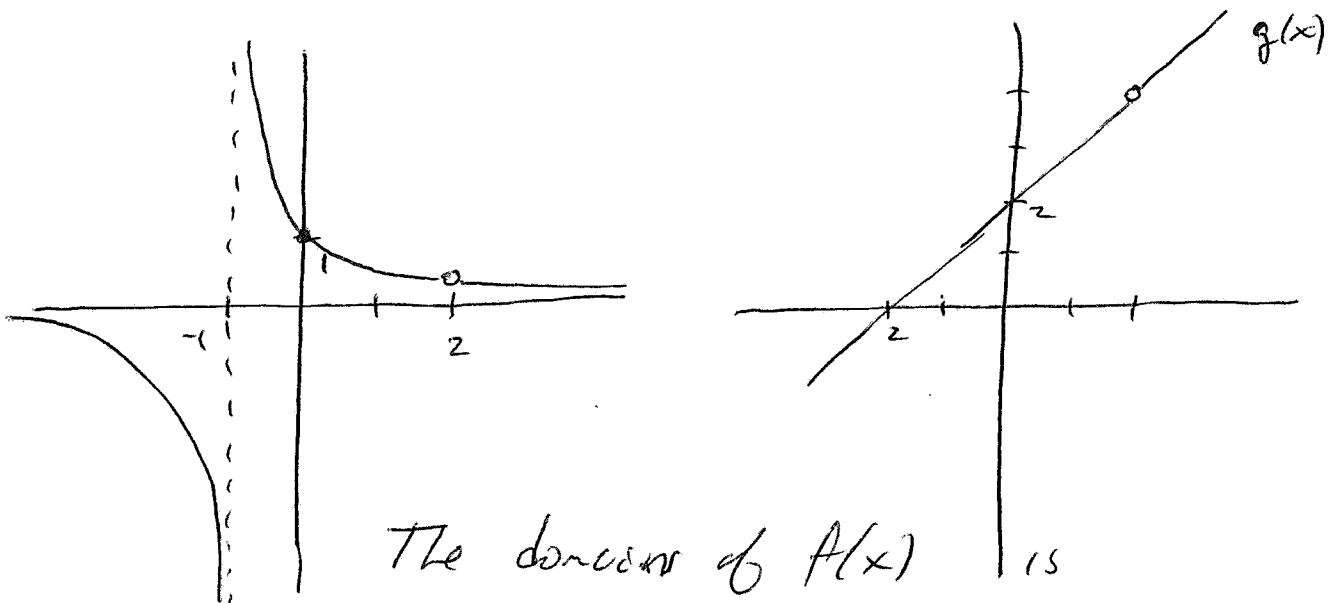
Class 8: ~~Section 3.2~~ Section 3.2 I

To start, here are 2 examples:

ex. $f(x) = \frac{x-2}{x^2-x-2}$ and $g(x) = \frac{x^2-4}{x-2}$.

Sketch their graphs and find $\lim_{x \rightarrow 2} f(x)$

$\lim_{x \rightarrow -1} f(x)$, and $\lim_{x \rightarrow 2} g(x)$, if they exist.



The domains of $f(x)$ is

$\{x \in \mathbb{R} \mid x \neq 2, -1\}$ and $g(x)$ is $\{x \in \mathbb{R} \mid x \neq 2\}$.

because of the divide by zero problem (rational functions where denominator is 0).

For $f(x)$, notice that as long as $x \neq -1, 2$,

$$f(x) = \frac{x-2}{x^2-x-2} = \frac{x-2}{(x-2)(x+1)} = \frac{1}{x+1}$$

is easy to see.

Caution, however, that the pt $x=2$ is still not in the domain of $f(x)$.

for $g(x)$, we see that as long as $x \neq 2$,

$$g(x) = \frac{x^2 - 4}{x - 2} = \frac{(x-2)(x+2)}{x-2} = x+2 \text{ on all } x \neq 2.$$

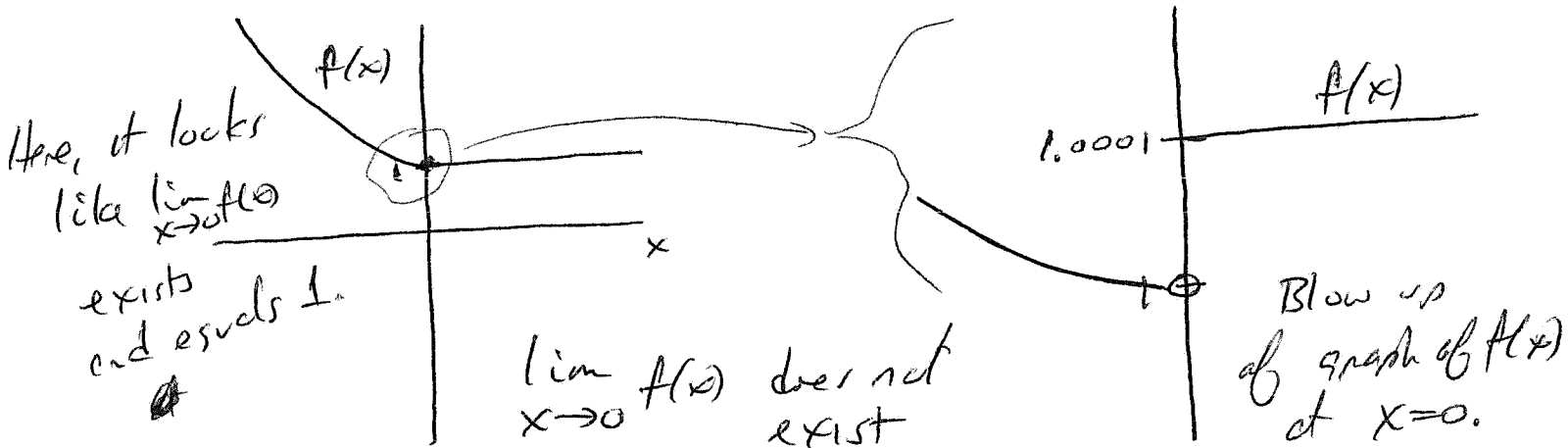
Now discuss whether the limits exist.

Section 3.1 uses tables and graphs to help determine limits

Visual aids are nice (calculators and computers) but be very careful. They can be tricky.

ex: $\lim_{x \rightarrow 0} \sin \frac{4\pi}{x}$ (ex 8, pg 97)

ex: $\lim_{x \rightarrow 0} f(x)$ where $f(x) = \begin{cases} e^{-x} & x < 0 \\ 1.0001 & x \geq 0 \end{cases}$



Even though limits may be tricky or difficult to discern in certain situations, most functions are well-behaved and have nice properties that are easy to identify.

One sided limits

Def A function $f(x)$ has a limit L "from the right" at $x=c$, denoted $\lim_{x \rightarrow c^+} f(x) = L$ if limit exists on the right side of c .

Note: Similar def "from the left," $\lim_{x \rightarrow c^-} f(x) = L$.

Thm Given $f(x)$ defined near a pt $c \in \mathbb{R}$ but possibly not at $x=c$, if $\lim_{x \rightarrow c^-} f(x) = a$ and $\lim_{x \rightarrow c^+} f(x) = b$, then

$$\lim_{x \rightarrow c} f(x) = L \quad \text{iff} \quad a = b = L.$$

Final notes

- ① The limit laws on pg 98 will be your primary means of finding limits of complicated functions once the easy ones are known
- ② For the elementary functions of Chapter 1: power fns, polynomials, rational functions, exponential fns, logarithmic, trigonometric for each pt c in the domain of function,

$$\lim_{x \rightarrow c} f(x) = f(c).$$

This is a special (and nice) property.

Def A function $f(x)$ is said to be continuous at $x=c$ if $\lim_{x \rightarrow c} f(x) = f(c)$.

Notes on continuity

- ① This is exactly the notion that ~~the~~ the graph of $f(x)$ can be drawn "through" on input value c without "lifting the pencil from the paper".
- ② All elementary functions of Chapter 1 are continuous on their domains.

Special Note: It may sound strange, but when restricted to its domain, a rational function will satisfy

$$\lim_{x \rightarrow c} f(x) = f(c) \text{ for every } c \text{ in its domain}$$

(the pts ~~of~~ of $f(x) = \frac{p(x)}{q(x)}$ where $q(x) = 0$ are NOT in the domain). Hence rational functions are continuous ~~at~~ for every pt in their domain.

ex. in the examples $f(x) = \frac{x-2}{x^2-x-2}$, $g(x) = \frac{x^2-4}{x-2}$

f and g are continuous functions on their domains.

③ A function $f(x)$ which is continuous at every pt in its domain is said to be a continuous function on that domain.

ex. $g(x) = \frac{x^2 - 4}{x - 2}$ is continuous on $\{x \in \mathbb{R} \mid x \neq 2\}$.

$h(x) = \begin{cases} g(x) & x \neq 2 \\ 1 & x = 2 \end{cases}$ is not continuous on \mathbb{R} .
why?

$i(x) = \begin{cases} g(x) & x \neq 2 \\ 4 & x = 2 \end{cases}$ is continuous on \mathbb{R} .
why?

④ Continuity needs three criteria:

① For $\lim_{x \rightarrow c} f(x) = f(c)$, we need

① c must be in domain of f

② $\lim_{x \rightarrow c} f(x)$ must exist

③ limit must equal $f(c)$.

⑤ If $f(x)$ is not continuous at $x=c$, then we say f is discontinuous at $x=c$.

⑥ What happens when a domain has an edge pt?

Def A function is continuous on the domain $[a, b]$ if

① $\lim_{x \rightarrow c} f(x) = f(c)$ for all $c \in (a, b)$

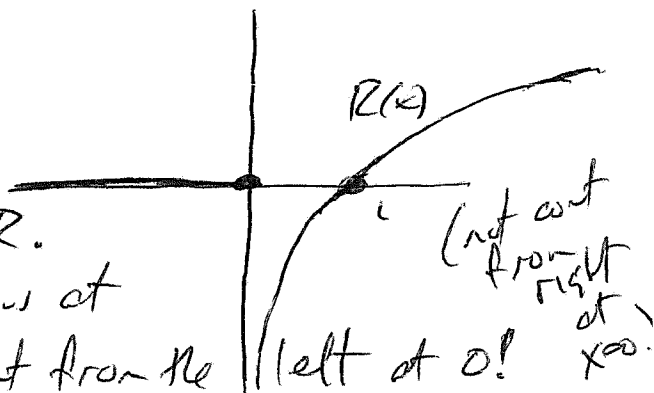
② $\lim_{x \rightarrow a^+} f(x) = f(a)$ called continuous from the right at a .

③ $\lim_{x \rightarrow b^-} f(x) = f(b)$ called continuous from the left at b .

ex. Given $f(x) = \ln x$, we have

- $f(x)$ is continuous on $[1, \infty)$
- $f(x)$ is continuous on $(0, a)$
- $f(x)$ is not continuous on $[b, a)$,
for any $b \leq 0$.

ex. $R(x) = \begin{cases} \ln x & x > 0 \\ 0 & x \leq 0 \end{cases}$



Domain of $R(x)$ is all \mathbb{R} .

But $R(x)$ is discontinuous at $x=0$, though cont from the left at 0!

⑦ The product, sum, difference, and ratio of 2 continuous functions is continuous (where ~~the~~ wherever the domain makes sense, that is).

ex. $p(x) = 1+x$ and $q(x) = x$ are continuous functions on all \mathbb{R} .

• So is $p(x) + q(x) = 1 + 2x$

• So is $p(x)q(x) = x + x^2$

• But $\frac{p(x)}{q(x)} = \frac{x+1}{x} = 1 + \frac{1}{x}$ is only continuous

on $\{x \in \mathbb{R} \mid x \neq 0\}$.