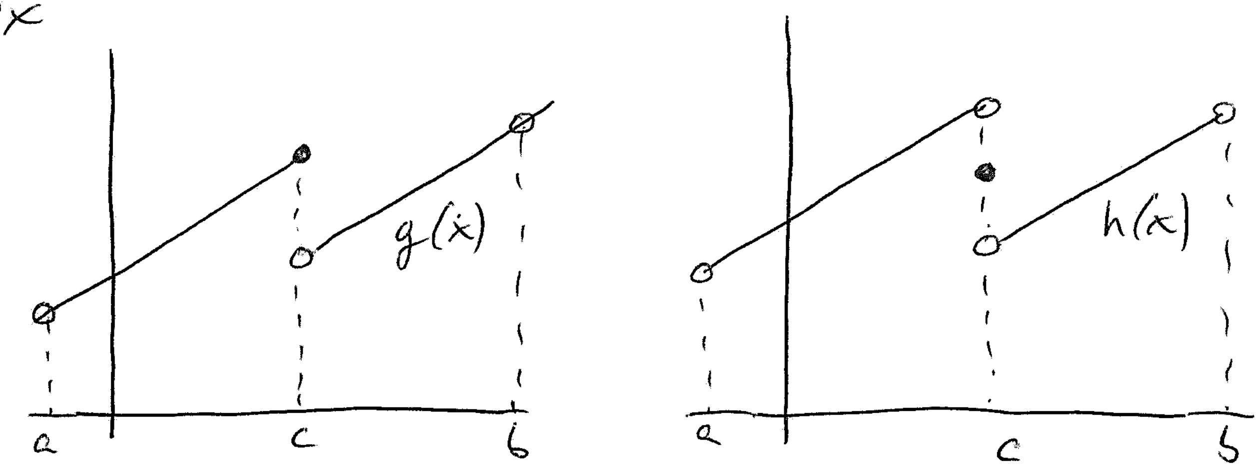
Class 22: Section 5.1

ex



- $g(x)$ has a local max at $x=c$ but no local min. (why not?) on (a,b).
- $h(x)$ has neither a local max or a local min anywhere on (a,b).

Note: In applications, local extrema are very important.

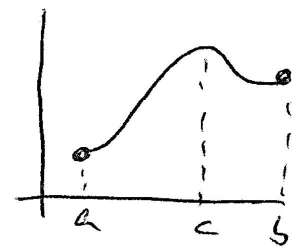
Q: How does one locate extrema? Where would they occur?

A: Think through the previous examples. By the EVT, $f(x)$ on a closed $[a,b]$ will have extrema at 2 possible places:

Extrema of $f(x)$ on $[a,b]$ occur at

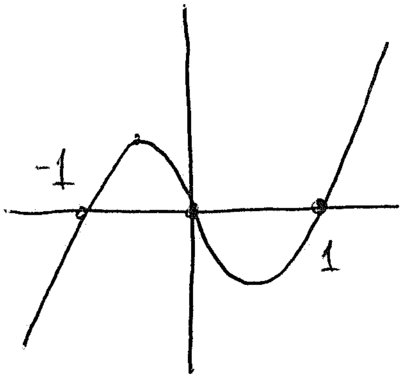
(I) An endpt, or

(II) An interior pt.



Here max at $x=c$, min at $x=a$.

ex. $f(x) = x(x-1)(x+1) = x(x^2-1) = x^3 - x$



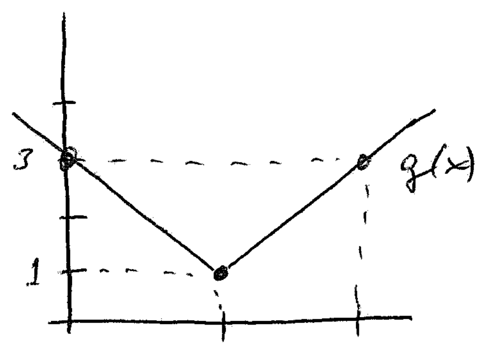
Q: How can we locate the only local max of $f(x)$?

A: $f(x)$ is diff. and max occurs where

$$f'(x) = 0 = \frac{d}{dx}[x^3 - x] = 3x^2 - 1.$$

This is solved by $x = \pm \frac{1}{\sqrt{3}}$

ex. $g(x) = 2|x-1| + 1$



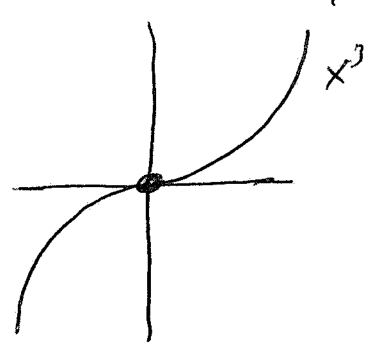
Here the derivative of $g(x)$ where it exists is never 0. However, there is a local min at $x=1$. What are the features of the graph that may help you locate the extrema?

Fermat's Thm (Not his last!).

If f has a local extremum at an interior pt c of an interval, and $f'(c)$ exists, then $f'(c) = 0$.

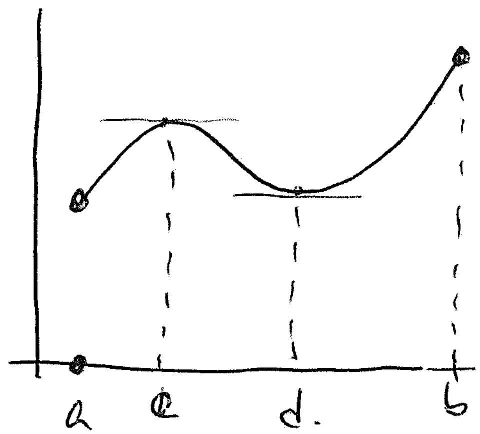
Notes ① The converse is not true; Just because there is a pt c, where $f'(c)$ exists and $f'(c) = 0$, it does not follow that there is an extremum at $x = c$.

ex. $f(x) = x^3$. $f'(0) = 3x^2\big|_{x=0} = 0$ and yet $x = 0$ is not an extremum.



② If $f(x)$ is diff on D, then a global extremum, if it exists, will occur either at an endpt of D or at an interior pt c where $f'(c) = 0$.

ex



Here, $h(x)$ is diff on (a,b) and cont on $[a,b]$. Hence all local extrema will be at four possible places: $x=a$, $x=b$, $x=c$, or $x=d$.

These are the 2 endpts and the 2 interior pts where $h'(x)=0$.

Q: What if there are a few places where $f(x)$ is not diff. Can they be extrema?

Def. A pt c in the domain of $f(x)$ where either $f'(c)=0$ or $f'(c)$ does not exist is called a critical pt of f.

Fact For $f(x)$ continuous on a closed, bounded interval, the global max will occur either at (1) an end pt, or (2) at a critical pt.

[illegible] Section 5.[illegible]

~~[illegible]~~

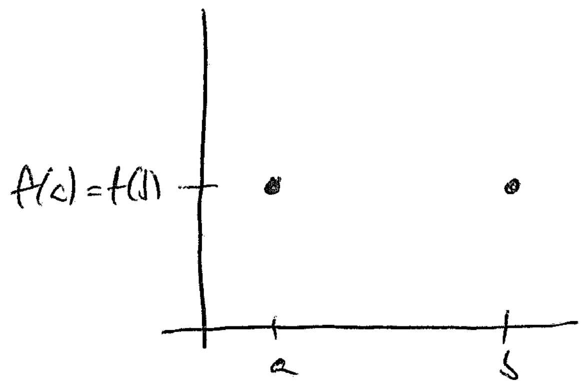
VII

2 big idea's

(I) Rolle's Thm

Suppose $f(x)$ is continuous on $[a,b]$ and diff. on (a,b), where $f(a) = f(b)$.

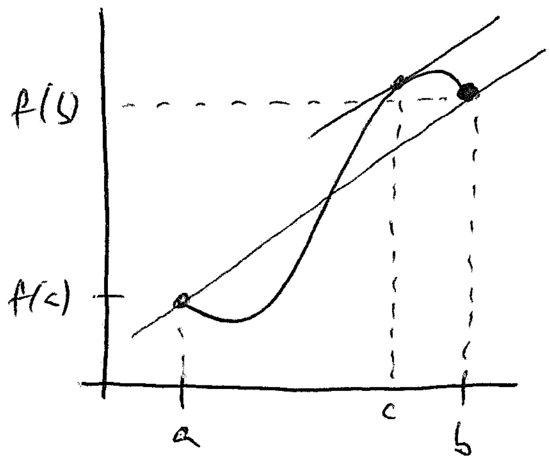
Then there exists a pt $c \in (a,b)$ where $f'(c) = 0$



pf. EVT says there must be a max. If it is in (a,b) then by Fermat, f' is 0 here. If it is at an end pt, then there must be a min. in the interior (why?). By Fermat again, derivative must be 0 at this pt ∎

VIII

<u>Mean Value Thm</u> [Inclined Rolle's Thm].



If $f(x)$ is continuous on $[a,b]$ and diff on (a,b), then there is a pt $c \in (a,b)$ where $f'(c) = \frac{f(b) - f(a)}{b-a}$

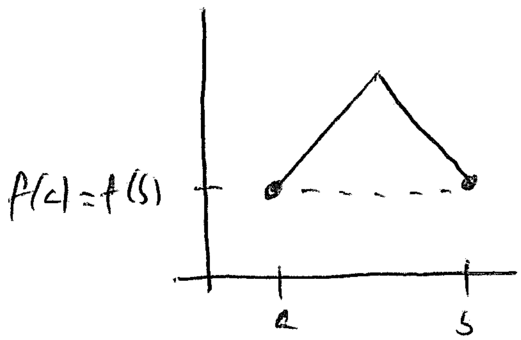
~~pf.~~ is straightforward and in the book.

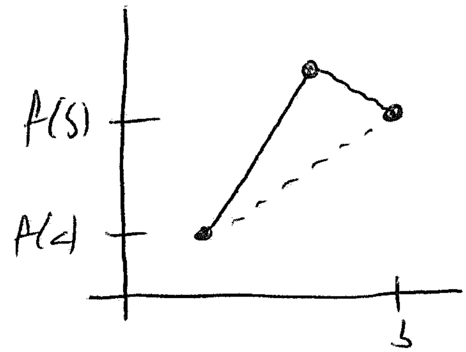
<u>Notes</u> ① In both of these thms, the average rate of change of $f(x)$ on $[a,b]$ is $\frac{f(b)-f(a)}{b-a}$. Thus both thms say that somewhere in (a,b), the instantaneous rate of change of $f(x)$ must equal the average rate of change on $[a,b]$.

Instantaneous rate of change at $c \in (a,b)$ | Average rate of change over all $[a,b]$.

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Notes ② MVT and Rolle's Thm do not work
cont'd. if f is not differentiable on (a,b)





avg rate of change on [a,b] is 0, but no place c, where $f'(c)=0$

average rate of change is $\frac{f(b)-f(a)}{b-a}$ but no place c where $f'(c)$ equals this quantity.

③ If you travel the entire New Jersey Turnpike (122 miles) in 90 minutes, it must be the case that at some point you were going $\frac{122 \text{ miles}}{1.5 \text{ hours}} = 81+$ miles/hour

Electronic Ticket, maybe (entry tickets, and EZPass are time-stamped).