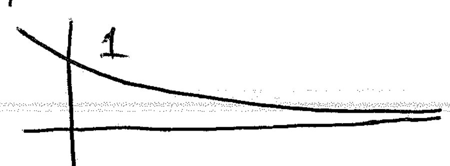
Class 23: Section 5.2

Application: The Bertelanffy equation models fish body size (fish never stop growing while alive).

Graph $L(x) = L_\infty - (L_\infty - L_0)e^{-kx}$, where $k > 0$, $L_\infty \geq L_0 > 0$.

Here, ⓐ e^{-kx}, $k>0$ look like

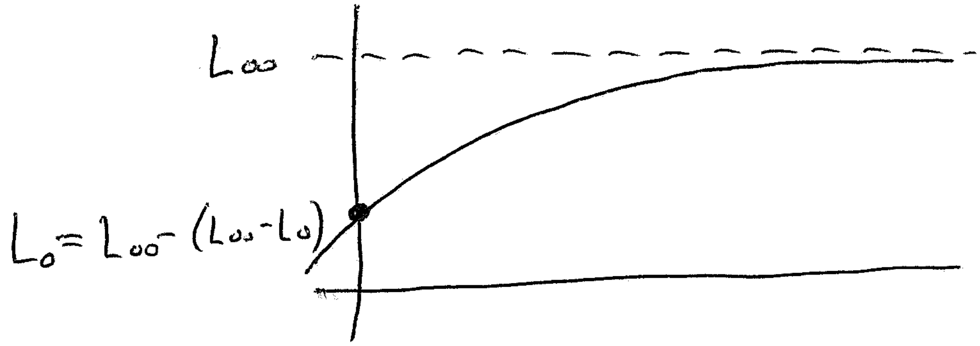


ⓑ $-e^{-kx}$ looks like

ⓒ $-Re^{-kx}$ looks like

$R = L_\infty - L_0 > 0$.

(d) So $L(x) = L_\infty - (L_\infty - L_0)e^{-kx}$



Q: What are the properties of this function?

<u>Def</u> A function $f(x)$ on an interval I is called (strictly) increasing on I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ on I

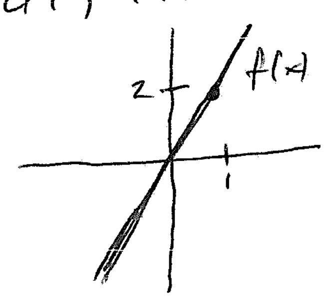
<u>Notes</u> (1) Similar definition for (strictly) decreasing.

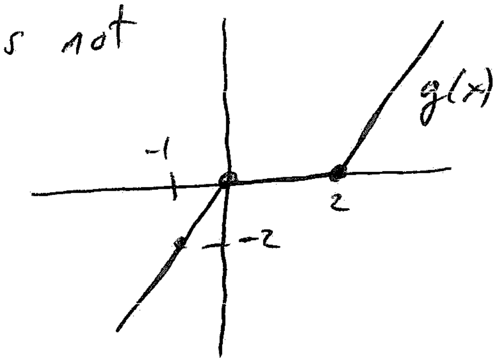
(2) Strictly means "$<$" as opposed to "$\leq$".

ex. $f(x) = 2x$ is strictly increasing on $[0, 4]$

$$g(x) = \begin{cases} 2x & x < 0 \\ 0 & 0 \leq x \leq 2 \\ 2x-4 & x > 2 \end{cases}$$

is not





③ Strictly increasing or strictly decreasing is also called <u>monotonic</u>.

④ A function like $g(x)$ above, where $g(x_1) \le g(x_2)$ for $x_1 < x_2$, is often called <u>non decreasing</u>, as it is not strictly increasing.

<u>Criteria for Monotonicity</u>

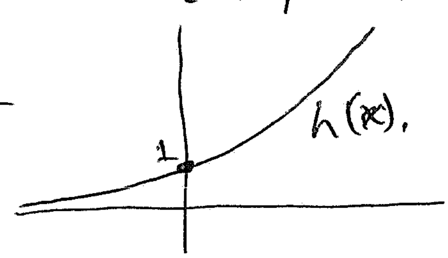
For $f(x)$ continuous on $[a,b]$ and differentiable on (a,b), we say

ⓐ if $f'(x) > 0$ for all $x \in (a,b)$, then $f(x)$ is increasing on $[a,b]$.

③ if $f'(x) < 0$ for all $x \in (a,b)$, then $f(x)$ is decreasing on $[a,b]$.

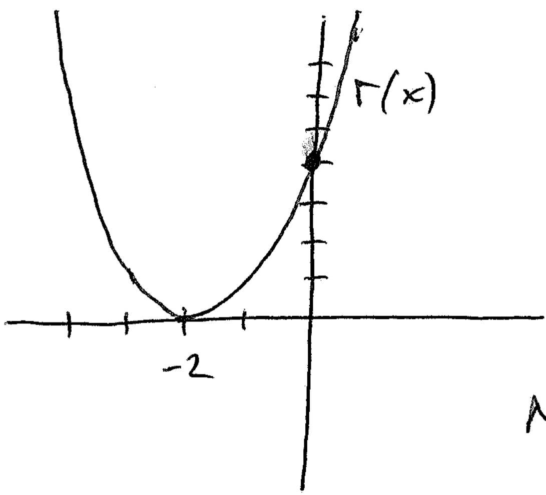
ex. $h(x) = e^{2x}$. Here $h'(x) = 2e^{2x} > 0$ everywhere on $\mathbb{R}$.

Hence $h(x)$ is increasing on $\mathbb{R}$



ex. $r(x) = x^2 + 4x + 4$. Here, $r'(x) = 2x + 4$.

Since $r'(x) > 0$ on $[-2, \infty)$, we can say that $r(x)$ is increasing on $[-2, \infty)$:



Also, $r'(x) < 0$ on $(-\infty, -2)$, so that $r(x)$ is decreasing on $(-\infty, -2]$.

Notice the graph.

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ex: $L(x) = L_\infty - (L_\infty - L_0)e^{-kx}$, $k > 0$, and $L_\infty > L_0 > 0$.

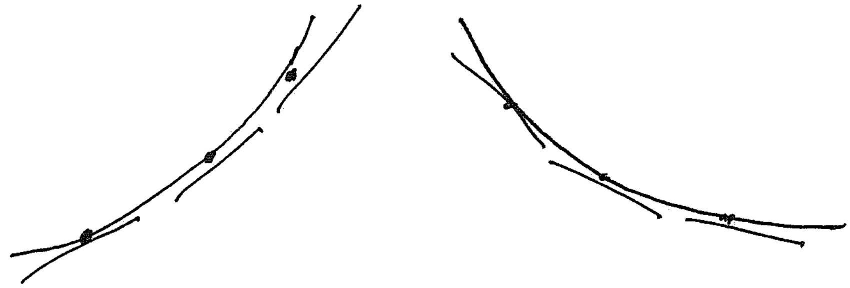
Here, $L'(x) = \underbrace{k}_{>0}\underbrace{(L_\infty - L_0)}_{>0}\underbrace{e^{-kx}}_{>0} > 0$

on $[0, \infty)$. Hence $L(x)$ is increasing on $[0, \infty)$.

So when $f'(x) > 0$, we know $f(x)$ is increasing.

Q: What does $f(x)$ look like when $f'(x)$ is increasing on an interval $[a,b]$?

(This means either $f(x)$ is negative and getting less so, or $f(x)$ is positive and getting bigger).



Facts ① For $f(x)$ diff, $f'(x)$ is just another function.

For $f'(x)$ diff., then $f''(x) = \frac{d}{dx}[f'(x)]$ is again another function.

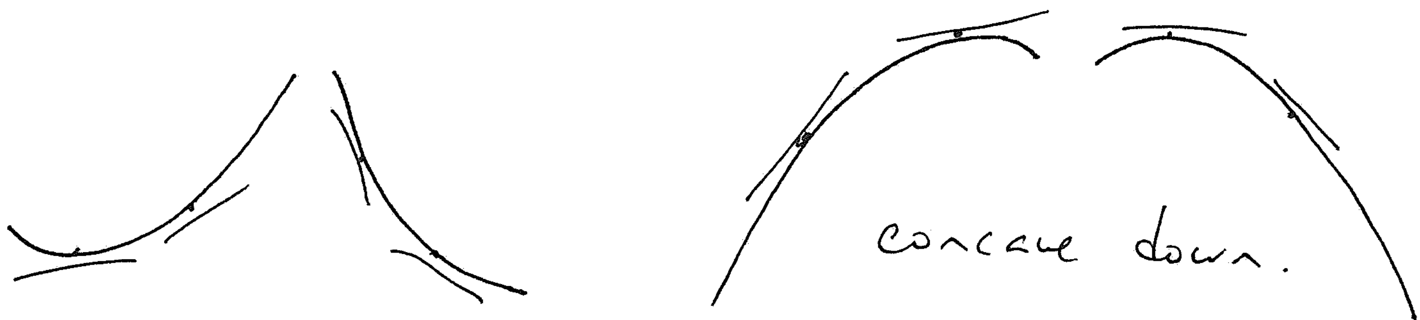
② $f''(x)$ measures how $f'(x)$ is changing. This, in turn, gives an idea on the "bending" of $f(x)$.

Def. For $f(x)$ twice diff. on I,

$f(x)$ is called

<u>concave</u> <u>up</u> if $f''(x) > 0$ on I,

<u>concave</u> <u>down</u> if $f''(x) < 0$ on I.



concave up

concave down.

ex. All quadratic polynomials are either concave up or down everywhere. (why?)

ex. $g(x) = e^x$ and $h(x) = e^{-x}$ are both concave up everywhere.

ex. $k(x) = x^3 - x$ is concave up on $[0,\infty)$ and concave down on $(-\infty,0]$ since $k''(x) = 6x$.