Lecture Questions I: 110.302 Differential Equations

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Determine the truth of the following two statements:

1. Every separable ODE can be written as an exact ODE.
2. Every linear ODE can be written as a separable ODE.

A. Both are true.
B. (1) is true and (2) is false.
C. (1) is false and (2) is true.
D. Both are false.
Question 2: Let $\dot{x} = x^2 - 2x + c$, for $c \in \mathbb{R}$.

For $c = -15$, the phase line for the ODE has the following characteristics:

A. A sink at $x = -3$ and a source at $x = 5$.
B. A source at $x = -3$ and a sink at $x = 5$.
C. A sink at $x = -5$ and a source at $x = 3$.
D. A source at $x = -5$ and a sink at $x = 3$.
E. Not enough information to tell.
Question 2: Let $\dot{x} = x^2 - 2x + c$, for $c \in \mathbb{R}$.

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B. A source at $x = -3$ and a sink at $x = 5$.
C. A sink at $x = -5$ and a source at $x = 3$.
D. A source at $x = -5$ and a sink at $x = 3$.
E. Not enough information to tell.

For $c = -15$ and $x(0) = 0$, $\lim_{t \to -\infty} x(t) =$

A. $-\infty$.
B. $\infty$.
C. The sink in your previous answer.
D. The source in your previous answer.
E. None of the above.
The ODE

\[
\left( \sin t + \sqrt{2-t} \right) y + e^{t \ln(t+1)} \left( t^2 - 10^{-2000} \right) y' = t^4
\]

is linear. By the Existence and Uniqueness Theorem for first-order linear ODEs, we know the following:

A. Solutions are not guaranteed to exist at all at the point \((0, 2) \in \mathbb{R}^2\).
B. Solutions are guaranteed to exist but may not be unique at \((0, 2)\).
C. A solution exists and is unique passing through \((0, 2)\).
D. There is not enough information to determine whether solutions exist and/or are unique at \((0, 2)\).
E. I have no idea.