

Lecture Questions II: 110.302 Differential Equations

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Question 1

In the Variation of Parameters Method for finding a particular solution to a nonhomogeneous, second-order linear ODE $L[y] = g(t)$, determine which one of the following is a true statement:

- A. The method only works when $g(t)$ has a certain type (a sum of products of exponentials, sines or cosines and polynomials.)
- B. The resulting system in the method,

$$u_1' y_1 + u_2' y_2 = 0, \quad u_1' y_1' + u_2' y_2' = g(t),$$

where $y_1(t)$ and $y_2(t)$ comprise the fundamental set of homogeneous solutions, may be inconsistent. In this case, $Y(t) = u_1 y_1 + u_2 y_2$ would not exist.

- C. It is possible for one of the two of $u_1(t)$ and $u_2(t)$ to be a constant as long as the other is not.
- D. One must check the Wronskian to see if $Y(t)$ is actually independent of y_1 and y_2 .
- E. None of the previous statements is true.

Question 2

Let

$$ay'' + by' + cy = 0, \quad y(t_0) = y_0, \quad y'(t_0) = y'_0$$

be a homogeneous, second-order, linear IVP with constant coefficients. If $a, b, c > 0$, what can we say about the long term behavior of the solutions?

- A. Solutions exist and are uniquely defined for all $t \in \mathbb{R}$, and tend to either ∞ or $-\infty$ as $t \rightarrow \infty$.
- B. Solutions exist and are uniquely defined for all $t \in \mathbb{R}$, and all tend to 0 as $t \rightarrow \infty$.
- C. Solutions exist and are uniquely defined only for certain choices of a , b , and c , and where these solutions tend to depends on the initial data.
- D. Solutions exist and are uniquely defined always, but there is not enough information to determine where solutions go.
- E. Not enough information available to determine whether solutions exist and/or are uniquely defined.

Question 3

Determine the truth of the following two statements about the Wronskian determinant of any two solutions to $y'' + p(t)y' + q(t)y = 0$:

- ① The Wronskian can be used to determine precisely where solutions to the ODE exist and are unique.
 - ② If one solution is already known, one can use the Wronskian determinant to construct another independent solution.
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- A. Both are true.
 - B. (1) is true and (2) is false.
 - C. (1) is false and (2) is true.
 - D. Both are false.