# Lecture 1: [redacted]

Start with $y = f(x)$, some unknown functional relation between 2 variables, where
$x$ – independent variable
$y$ – dependent variable

Such an equation is (or sets of them are) called a <u>mathematical</u> <u>model</u> when the variables represent measurable quantities in some application (usually set up to study some unknown entity $y$ based on its relationship to something controllable $x$).

More general def than in text (pg. 2).

If $y = f(x)$ is known, then we can simply study its properties (using calculus).

Often, though, we do not know $y = f(x)$, but we do have information about some properties, like derivatives, for example.

examples

(I) $\frac{dy}{dx} = ky, \quad k \in \mathbb{R}.$

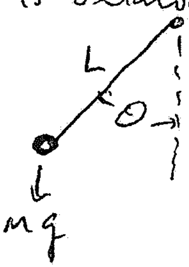
(II) $F = ma$ (Newton's 2nd Law of motion).

(III) $f'(x) = x - e^{x/2}$ (restatement of: Find $\int (x - e^{x/2})\,dx$).

or $y' = g(x)$, $y(x)$ is unknown soln: $y(x) = \int g(x)\,dx$

(IV) $$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$$

The Pendulum.




The above are mathematical models where the actual function is only known implicitly....

**Def** An ordinary differential equation (ODE) is an equation involving an unknown function between 2 entities and some of its derivatives.

Note: "Ordinary" means that the unknown function is a function of one independent variable.

ex. The Heat Equation (in 3-space)

$$\frac{du}{dt} = \alpha\left(\frac{d^2u}{dx^2} + \frac{du^2}{dy^2} + \frac{du^2}{dz^2}\right)$$

is a partial diff. eqn since $u$ is a fnc of more than 1-indep. variable.

Data-ODE pg 19.

Def. The order of an ODE is the same as the order of the highest derivative that appears in the equation:

pg 20

ex ① and ③ are first order ODEs.
② and ④ are 2nd order ODEs (do you see this?)

Def The general form of an nth order ODE is

$$F(x, y, y', \dots, y^{(n)}) = 0 \qquad (*)$$

pg 20

where $x$ – indep var (time?)
$y$ – dep var (unknown fnc $y = f(x)$)
$y^{(i)}$ is the ith derivative of $y = f(x)$
$F$ is some expression in $x, y, y', \dots, y^{(n)}$

Note: Sometimes, we can solve for the highest derivative:

Not quite refined in text ~~(**)~~ eqn 80 pg 20

$$y^{(n)} = h(x, y, y', \cdots, y^{(n-1)}), \qquad (**)$$

But not always:

ex. $y^{(5)} + \sin y^{(5)} = y^{(3)}$ cannot be written like $(**)$. But for $(*)$, $F = y^{(5)} + \sin y^{(5)} - y^{(3)}$.

---

Def. A function $f(\underbrace{x_1, \cdots, x_n}_{\vec{x}}, y_1, \cdots, y_m)$, or eqn $f(\vec{x}, \vec{y}) = c$

is linear in the variables $y_1, \cdots y_m$ if

$$f(x_1, \cdots, x_n, y_1, \cdots, y_m) = a_0(\vec{x}) + \sum_{i=1}^{m} a_i(\vec{x}) y_i$$

Not quite in text

where the $a_i(x)$, $i = 0, \cdots, m$ are arbitrary.

Notes ① For an ODE to be linear, it must be linear in $y, y', \cdots, y^{(n)}$, and can be written as

in text pg 21

$$a_n(\vec{x}) y^{(n)} + \cdots + a_1(\vec{x}) y' + a_0(\vec{x}) y = g(x).$$

examples

Ⓐ $(\sin x)y' + (\ln x)y = \tan e^x$ is a linear $1^{st}$ order ODE.

Ⓑ $y'' + xy' + \sin y = 0$ is Not linear

Ⓒ $y''y + y' = 0$ is Not linear.

---

Suppose $y' = f(t,y)$ is a $1^{st}$ order linear ODE (written like (**)).

Then there exist functions $p(t), q(t)$ so that $f(t,y) = -p(t)y + q(t)$, and the ODE can be written as

$$y' + p(t)y = q(t)$$

This form will be very important to understanding how to study this type of ODE.

ex. Given $(\sin x)y' + (\ln x)y = \tan e^x$, identify $p(t)$.

Solution: Divide by $\sin x$ to get,

$$y' + \left(\frac{\ln x}{\sin x}\right)y = \frac{\tan e^x}{\sin x}. \quad p(t) = \frac{\ln x}{\sin x}.$$

Def. A solution to $(*)$ on $I=(a,b)\subset\mathbb{R}$ is any function $y=f(x)$ that satisfies the equation $(*)$.

pg 22.

ex. $\frac{dp}{dt}=\frac{p}{2}-450$ is solved by $p(t)=900+ce^{t/2}$ $\forall c\in\mathbb{R}$.

- How do we know? Try it.
- How did we find it? Keep listening.
- What to make of the parameter c?

Hint: This is a calculus consequence, after all!

ex. Sometimes a solution is only known implicitly:

Show $x^2+y^2-5=0$ solves $\frac{dy}{dx}=-\frac{x}{y}$.

Here only locally can we solve for $y(x)$.

ex. Solve $x''(t)+\frac{k}{m}x(t)=0$.

ex Solve $y'(x)=x-e^{x/2}$