110.302 Lecture 11:

Before talking about the cases where roots of the characteristic equation are the same or not real, lets return to the more general linear 2^{nd} order homogeneous ODE

$$y'' + p(t)y' + q(t)y = 0$$

To study this, form the operator (an operator is a function whose domain and range are functions)

$$L[\varphi] = \varphi'' + p(t)\varphi' + q(t)\varphi.$$

This operator is defined for all C^2 functions $y(t)$ on an interval like $\alpha < t < \beta$, where α, may be a number or $-\infty$, and β may be a number or ∞.

Notes ① Can also write

$$L = \frac{d^2}{dt^2} + p\frac{d}{dt} + q$$

② An operator $L[\varphi]$ is linear if

$$L[c_1\varphi_1 + c_2\varphi_2] = c_1 L[\varphi_1] + c_2 L[\varphi_2]$$

Claim: $L[\varphi] = \varphi'' + p(t)\varphi' + q(t)$ is linear. as an operator.

pf
$$L[c_1\varphi_1 + c_2\varphi_2] = \frac{d^2}{dt^2}[c_1\varphi_1 + c_2\varphi_2] + p(t)\frac{d}{dt}[c_1\varphi_1 + c_2\varphi_2] + q(t)(c_1\varphi_1 + c_2\varphi_2)$$

$$= c_1\varphi_1'' + c_2\varphi_2'' + p(t)(c_1\varphi_1' + c_2\varphi_2') + q(t)(c_1\varphi_1 + c_2\varphi_2)$$

$$= c_1(\varphi_1'' + p(t)\varphi_1' + q(t)\varphi_1) + c_2(\varphi_2'' + p(t)\varphi_2' + q(t)\varphi_2)$$

$$= c_1 L[\varphi_1] + c_2 L[\varphi_2].$$

<u>Fact</u>: The homogeneous 2^{nd} order linear ODE

$$y'' + p(t)y' + q(t)y = 0$$

is solved by <u>any</u> function $y(t)$, where

$$L[y(t)] = 0.$$

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2 Theorems on linear, ~~[illegible]~~ 2^{nd} order ODEs.

(I) Existence & Uniqueness

<u>Thm</u> The IVP $y'' + p(t)y' + q(t)y = g(t)$, $y(t_0) = y_0$, $y'(t_0) = y_0'$, where $p, q,$ and g are continuous on an open interval I containing t_0, has a unique solution $y(t)$ defined and twice differentiable on I.

<u>Note</u>: Here, I can be taken to be the largest interval containing t_0 where $p, q,$ and g are all simultaneously continuous.

(II) Superposition ~~Thm~~

<u>Thm</u> If $y_1(t), y_2(t)$ are 2 solutions to $L[y]=0$, then so is $c_1 y_1 + c_2 y_2$ for all $c_1, c_2 \in \mathbb{R}$.

<u>Caution</u>: Unless y_1 and y_2 are chosen carefully, there may be other solutions. We need to know what <u>carefully</u> means here.