110.302 Lecture 22:

XIV

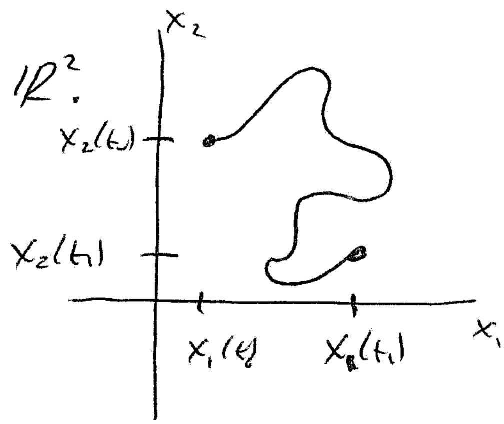
Homogeneous with constant coefficients

let $\vec{x}' = A_{n\times n}\vec{x}$, $A_{n\times n}$ - matrix of constants.

(note: $\vec{x}' = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\vec{x}$ is an example).

For $n=2$, solutions look like $\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$.

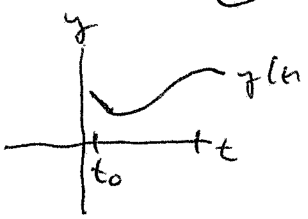
For each t, $\vec{x}(t)$ is a pt in $\mathbb{R}^2$.
As t evolves, $\vec{x}(t)$ will trace out a parametrized curve

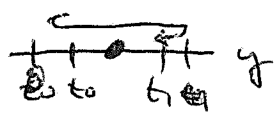


We call the x_1x_2-plane the phase plane for the system, noting that

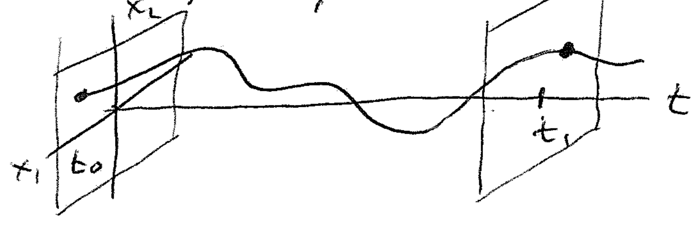
ⓐ the independent variable t is implicit to the graph (not an axis but on the curve)

ⓑ For one equation $y' = f(y)$, the solution $y(t)$ lived in the ty-plane, but we could also track its evolution in the phase line





ⓒ We could graph $\vec{x}(t)$ in the tx_1x_2-space, but this is hard to see

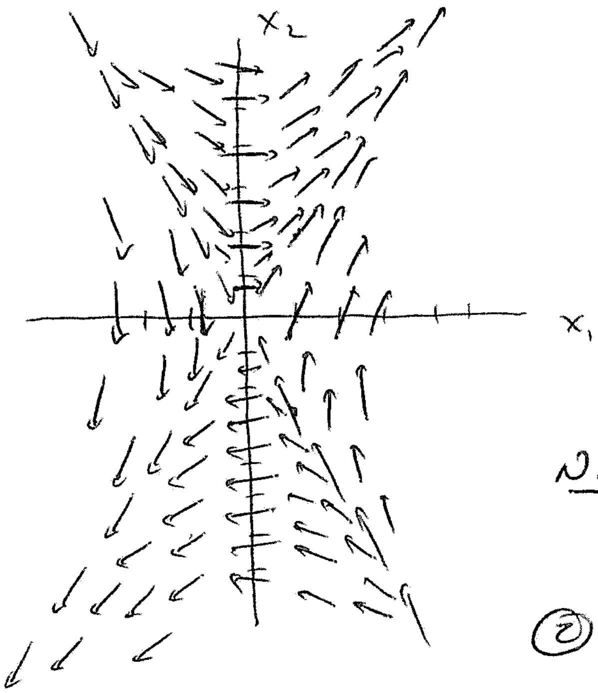


see overhead for example

Some ideas for study

① $\vec{x}'(t) = A\vec{x}(t)$, by simply choosing pts $\vec{x} \in \mathbb{R}^2$, we can plot tangent lines, and make a slope field in the phase plane

ex. Let $A = \begin{bmatrix} 1 & 1 \\ 6 & 0 \end{bmatrix}$. Compute (a) $\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{x}' = \begin{bmatrix} 1 & 1 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$.



(b) $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{x}' = \begin{bmatrix} 1 & 1 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$

(c) $\vec{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\vec{x}' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ over, up

⋮

<u>Notes</u> ① looks very similar to example 1 pg 358. (420?)

② Use JODE 2D calculator on website

Ⅱ The solution curves will be the integral curves of this slope field:

(a) Given a value of $c_1, c_2 \in \mathbb{R}$, the curve $\vec{x}(t) = c_1\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{3t} + c_2\begin{bmatrix} 1 \\ -3 \end{bmatrix}e^{-2t}$ will be one of those curves.

(b) Straight line motion occurs when c_1 or $c_2 = 0$.

choose $c_1 = -2$, $c_2 = 0$. then

$\vec{x}(t) = -2\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{3t} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}e^{3t}$