HOMEWORK PROBLEM SET 1: DUE FEBRUARY 10, 2017

110.302 DIFFERENTIAL EQUATIONS
PROFESSOR RICHARD BROWN

Question 1. For the following, determine the order of the ODE and whether the ODE is linear or nonlinear. Justify your conclusions by explanation.

(a) $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} - \frac{d^2y}{dt^2}\frac{dy}{dt} + y = 1.$

(b) $\frac{dy}{dt} + y\sin^2 t = 0.$

(c) $\frac{d^3y}{dt^3} + \sin(t+y) = e^t.$

(d) $(\ln t)\frac{d^2y}{dt^2} + \frac{1}{t}\frac{dy}{dt} = t^2 y.$

Question 2. For the following, verify that the given functions are solutions to the ODE.

(a) $2t^2y'' + 3ty' - y = 0,\ t > 0; \quad y_1(t) = \sqrt{t}, \quad y_2(t) = t^{-1}.$

(b) $y'' + y = \sec t,\ 0 < t < \frac{\pi}{2}; \quad y = (\cos t)\ln\cos t + t\sin t.$

(c) $y'' = a\sqrt{1 + (y')^2}; \quad y = \frac{e^{at}+e^{-at}}{2a}.$

Question 3. For the following, determine the values of r for which the given differential equation has solutions of the form given.

(a) $2y'' - 12y' + 10y = 0; \quad y(t) = e^{rt}.$

(b) $t^2y'' + 2ty' - 6y = 0,\ t > 0; \quad y(t) = t^r.$

Question 4. Do the following for the differential equation

$$y' = -ay + b,$$

for a and b positive numbers. (Note that this follows closely from the example that we started at the end of the second lecture. But now finish the calculations.)

(a) Solve the ODE. (That is, find the *general* solution.)

(b) Sketch the solution for several different initial conditions.

(c) Describe how solutions change when (1) a increases, (2) b increases, and (3) both a and b increase, but the ratio $\frac{b}{a}$ stays the same.

For the next two problems, we did not yet talk about slope fields in class (it is in Lecture 2 at the end but I am behind a bit in class. Still, Section 1.1 gives a good overview of what a slope field is and how to construct one. Use Section 1.1 and my notes from Lecture 2 to answer the following.

Question 5. Do text problems 1.1.15-1.1.20 (this is a quick matching exercise to help develop your intuition).

Question 6. In each of the ODEs below, draw a direction field (you can use technology). Based on the direction field, determine and describe the behavior of solutions $y(t)$ as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, then describe the dependency.

(a) $y' = 4 - 3y$.
(b) $y' = 4y - 3$.
(c) $y' = -y(2 - y)$.
(d) $y' = y + 2 - t$.