HOMEWORK PROBLEM SET 2: DUE SEPTEMBER 14, 2018

110.302 DIFFERENTIAL EQUATIONS
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Question 1. Solve the following linear differential equations (for the general solution if it does not have initial data, or the particular solution if it is an IVP).

(a) $2y' + y = 3t^2$.

(b) $y' + 2y = te^{-2t}, \quad y(1) = 0.$

(c) $ty' + (t+1)y = t, \quad t > 0, \quad y(\ln 2) = 1.$

Question 2. Solve the linear IVP

$$ty' + (t+1)y = 2te^{-t}, \quad y(1) = a, \quad t > 0,$$

and locate the particular value of $a = a_0$ for which the transition from one type of behavior to another occurs. Describe the behavior of the solution corresponding to a_0, as well as what happens to solutions for values of a on either side of this value a_0.

Question 3. For the IVP,

$$y' + \frac{2}{3}y = 1 - \frac{t}{2}, \quad y(0) = y_0,$$

find the value of y_0 for which the solution touches, but does not cross, the t-axis.

Question 4. Solve the following separable differential equations by separating the variables

(a) $y' = \dfrac{2x}{1+2y}$, $y(0) = 2$. (Note: I want an explicit expression for $y(x)$ here.)

(b) $xy' - y = 2x^2y$, for $x > 0$. (Note: This ODE is also linear.)

(c) $yy' = -2t(1+y^2)$, $y(0) = 1$.

Question 5. Determine where the solution to the IVP $y' = xy^2 + 2y^2$, $y(0) = 1$ attains its minimum value.