HOMEWORK PROBLEM SET 4: DUE SEPTEMBER 28, 2018

110.302 DIFFERENTIAL EQUATIONS
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Question 1. Verify that the following differential equations are exact and then solve:

(a) $(y-x)e^x + (1+e^x)\dfrac{dy}{dx} = 0, \quad y(1) = 1.$

(b) $\dfrac{dy}{dx} = -\dfrac{3x+4y}{4x-2y}, \quad y(0) = 2.$

(c) $y\cos(xy) - \tan x + x\cos(xy)y' = 0.$

Question 2. Find a value of the constant a that renders the ODE

$$a(ye^{2xy} + x) - (2xe^{2xy} + \cos y)y' = 0$$

exact. Then solve it.

Question 3. Verify that the ODE

$$x^2y^3 + x(1+y^2)y' = 0$$

is not exact, but via multiplication by the function $\mu(x,y) = \frac{1}{xy^3}$, the new ODE is exact. Then solve the new ODE and verify that the solution also solves the original ODE.

Question 4. Show that any separable, first-order differential equation is exact.

Question 5. Construct a bifurcation diagram for the ODE $y' = y^2 - (a-1)y$. Then determine the long term behavior of the solution to the ODE that satisfies $y(0) = 2$. Note that I have not told you the value of the parameter a, so you will have to classify the long-term behavior of your solution for all possible values of a.

Question 6. Consider the population model for a species of fish in a lake

$$\frac{dP}{dt} = 2P - \frac{P^2}{50},$$

where P is measured in thousands of fish and t is measured in years. The US Fish and Wildlife Service, which is managing the lake, wants to issue fishing licenses for the harvesting of some of the fish (this amounts to a constant term being subtracted off of the right hand side above, which is a function of h, the number of licenses issued). Each fishing license is valid for the annual take of 3000 fish. Draw a bifurcation diagram for the above ODE with the added parameter part, and answer the following questions.

(a) What is the largest number of licenses that can be issued if the goal is to keep a stable population of fish in the lake over the long term?

(b) If the largest number of licenses is actually issued, what is the expected long term stable population of fish in the lake?

(c) Solve the IVP given by the above differential equation and the initial value $P(0) = 2$ (This corresponds to an initial population of 2000 fish in the lake, and an assumption that there will be no harvesting, $h = 0$).

(d) As an expert consultant to the USFWS, discuss the ramifications of issuing the maximal number of licenses allowed by a mathematical model in the presence of real world issues which may temporary affect populations (drought, flooding, unlawful fishing, pollution, etc.)

(e) What is your final recommendation, in terms of the number of licenses that should be issued, to the USFWS? Back this final recommendation up with sound reasoning.