HOMEWORK PROBLEM SET 5: DUE OCTOBER 5, 2018

110.302 DIFFERENTIAL EQUATIONS
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Question 1. Suppose the IVP $y'' + p(t)y' + q(t)y = 0$, $y(t_0) = y_0$, $y'(t_0) = y'_0$ is solved by the two functions $y_1(t)$ and $y_2(t)$ and the general solution is

$$y(t) = c_1y_1(t) + c_2y_2(t).$$

Calculate c_1 and c_2 in terms of the initial data y_0, y'_0, $y_1(t_0)$, $y'_1(t_0)$, $y_2(t_0)$, and $y'_2(t_0)$. (Hint: I already gave you the result in class.)

Question 2. Solve the following:

(a) $2y'' + 5y' + 2y = 0, \quad y(0) = -1, \quad y'(0) = 5.$

(b) $y'' - 5y' + 5y = 0.$

(c) $y'' = \dfrac{1}{6}(y + y').$

(d) $4y'' = 3y', \quad y(0) = \dfrac{5}{2}, \quad y'(0) = -2.$

Question 3. Construct a second-order, linear, homogeneous, IVP with constant coefficients whose particular solution is $y(t) = 4e^{3t} - e^{-2t}$.

Question 4. Solve the IVP $32y'' - 2y = 0$, $y(0) = 4$, $y'(0) = \alpha$, and find the unique value of $\alpha \in \mathbb{R}$ so that $\lim_{t\to\infty} y(t) = 0$.

Question 5. Find the maximum value of a function $y(t)$ that satisfies the following: (1) its value at $t = 0$ is 3, (2) its derivative at $t = 0$ is $-\frac{2}{3}$, and (3) the function is the difference of four times its first derivative and three times its second derivative.

Question 6. Calculate the Wronskian of the following pairs of functions and determine all intervals where the Wronskian function $W(f, g)(x)$ is non-zero.:

(a) $f(x) = xe^{r_1x}, \quad g(x) = xe^{r_2x}.$

(b) $f(x) = \cos^2 x, \quad g(x) = 1 + \cos 2x.$

(c) $f(x) = x^2 + 1, \quad g(x) = 2x.$

Question 7. Verify that $y_1(t) = 1$ and $y_2(t) = \sqrt{t}$ both solve the ODE $yy'' + (y')^2 = 0$, for $t > 0$, but $y(t) = c_1 + c_2\sqrt{t}$ is not a general solution to the ODE. Explain why this result does not contradict Theorem 3.2.2 in the text on the Principle of Superposition.

Question 8. Determine the Wronskian of any two solutions to the ODE $t^2y'' - t(t+2)y' + (t+2)y = 0$ without actually solving the ODE.

Question 9. If the Wronskian of two functions $f(x)$ and $g(x)$ is x^2e^{2x}, and $g(x) = x$, then what is $f(x)$?