Question 1. Use Euler’s formula to do the following:

(a) Write \( e^{i\pi} \) in the form \( a + ib \).
(b) Write \( 2^{2-2i} \) in the form \( a + ib \).
(c) Write \( \pi^{2i-1} \) in the form \( a + ib \).
(d) Show that \( \cos t = \frac{e^{it} + e^{-it}}{2} \), and \( \sin t = \frac{e^{it} - e^{-it}}{2i} \).

Question 2. Solve the following:

(a) \( y'' - 6y' + 9y = 0, \quad y(0) = 0, \quad y'(0) = 2. \)
(b) \( 9y'' + 9y' - 4y = 0. \)
(c) \( y'' + 2y' + 6y = 0, \quad y(0) = 2, \quad y'(0) = 4. \)
(d) \( 2y'' + 2y' + y = 0. \)
(e) \( 2y'' + 2y' = 0. \)
(f) \( y'' = -a (ay + 2y'), \quad a \in \mathbb{R}. \)

Question 3. For the ODE \( ay'' + by' + cy = 0, \) suppose \( b^2 - 4ac < 0 \), so that the two solutions to the characteristic equation for the ODE are \( \lambda \pm i\mu \). Do the following:

(a) Show that \( u(t) = e^{\lambda t} \cos \mu t \) and \( v(t) = e^{\lambda t} \sin \mu t \) are, in fact, solutions to the ODE.
(b) Show that the Wronskian \( W(u(t), v(t)) = \mu e^{2\lambda t} \), thereby establishing the linear independence of \( u(t) \) and \( v(t) \) on all of \( \mathbb{R} \).
(c) Do the same for the ODE in the case that \( b^2 = 4ac \) by showing that \( u(t) = e^{-\frac{b}{2a}t} \) and \( v(t) = te^{-\frac{b}{2a}t} \) each solve the ODE and are linearly independent of each other as functions.

Question 4. For the IVP \( y'' = -\frac{4y-12y'}{9}, \quad y(0) = \alpha > 0, \quad y'(0) = -1, \) do the following:

(a) Find all value(s) of \( \alpha \) where \( \lim_{t \to \infty} y(t) = 0. \)
(b) Find all value(s) of \( \alpha \) where \( y(t) > 0, \) for all \( t \in \mathbb{R}. \)

Question 5. Given the following ODEs and the one solution given, write out a full fundamental set of solutions:

(a) \( t^2y'' + 2ty' = 2y, \quad t > 0, \quad y_1(t) = t. \)
(b) \( (x - 1)y'' - xy' + y = 0, \quad x > 1, \quad y_1(x) = e^x. \)
Question 6. Solve the following using the Undetermined Coefficients Method:

(a) $y'' + 4y = t^2 + 3e^t$.

(b) $\ddot{x} + 2\dot{x} + 5x = 4e^{-t} \cos 2t$, $x(0) = 1$, $\dot{x}(0) = 0$.

(c) $y'' - y' - 2y = \cosh 2x$. Hint: Write the hyperbolic cosine function as a linear combination of exponentials.