Question 1. Determine whether the set
\[ \left\{ (x, y) \in \mathbb{R}^2 \mid -1 < x < 1 \right\} \cup \left\{ (x, y) \in \mathbb{R}^2 \mid x = 2 \right\} \]
is open, closed, or neither.

Question 2. Evaluate the following limits, or show that they fail to exist:
(a) \[ \lim_{(x,y) \to (0,0)} \frac{e^x e^y}{x + y + 2}. \]
(b) \[ \lim_{(x,y) \to (0,0)} \frac{x^4 - y^4}{x^2 + y^2}. \]
(c) \[ \lim_{(x,y) \to (0,0)} \frac{x^2 - xy}{\sqrt{x^2 + y^2}}. \]

Question 3. Determine whether the functions are continuous on their domain:
(a) \[ f(x, y) = \begin{cases} \frac{x^2 - y^2}{x + y} & \text{if } (x, y) \neq (0,0) \\
0 & \text{if } (x, y) = (0,0). \end{cases} \]
(b) \[ g(x, y) = \begin{cases} \frac{x^3 + xy^2 + 2x^2 + 2y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0,0) \\
\frac{2}{y^2} & \text{if } (x, y) = (0,0). \end{cases} \]

Question 4. For \( f(x, y) = 2x - 10y + 3 \), do the following:
(a) Show that if \( \|(x, y) - (5, 1)\| < \delta \), then \( |x - 5| < \delta \) and \( |y - 1| < \delta \).
(b) Use the previous part to show that if \( \|(x, y) - (5, 1)\| < \delta \), then \( |f(x, y) - 3| < 12\delta \).
(c) Show that \( \lim_{(x,y) \to (5,1)} f(x, y) = 3 \).

Question 5. Do the steps below to establish that
\[ \lim_{(x,y) \to (0,0)} \frac{x^3 + y^3}{x^2 + y^2} = 0. \]
(a) Show that \( |x| \leq \|(x, y)\| \), and \( |y| \leq \|(x, y)\| \).
(b) Show that \(|x^3 + y^3| \leq 2(x^2 + y^2)^{3/2}\). (Hint: Begin with the Triangle Inequality and then use part (a).)

(c) Show that if \(0 < ||(x, y)|| < \delta\), then \(\frac{x^3+y^3}{x^2+y^2} < 2\delta\).

(d) Now prove that \(\lim_{(x,y) \to (0,0)} \frac{x^3+y^3}{x^2+y^2} = 0\).

**Question 6.** Calculate the partial derivatives of the following:

(a) \(f(x, y) = \frac{x^3-y^2}{1+x^2+3y^2}\).

(b) \(g(x, y) = \ln \left(\frac{x}{y}\right)\).

(c) \(F(x, y, z) = \sin(x^2y^3z^{-4})\).

**Question 7.** Show that one can rewrite the expression for the derivative of \(f : \mathbb{R} \to \mathbb{R}\) at \(x = a\), namely

\[
f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}, \quad \text{to} \quad \lim_{x \to a} \frac{f(x) - f(a) - f'(a)(x - a)}{x - a} = 0.
\]

This allows us to define the derivative of \(f\) at \(a\) through the existence of a linear function \(h : \mathbb{R} \to \mathbb{R}\), \(h(x) = f(a) + f'(a)(x - a)\) which makes the latter limit equation true.