Question 1. For the functions \( f \) and points \( a \) indicated, calculate \( Df(a) \):

(a) \( f(x, y, z) = (2x - 3y + 5z, x^2 + y, \ln(yz)) \), and \( a = (3, -1, -2) \).

(b) \( f \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} x^2y \\ x + y^2 \\ \cos \pi xy \end{bmatrix} \), and \( a = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \).

Question 2. Explain fully why the function \( f(x, y) = \left( \frac{xy^2}{x^2 + y^2}, \frac{x}{y} + \frac{y}{x} \right) \) is differentiable at every point of its domain.

Question 3. Find equations for the following spaces:

(a) All planes tangent to \( z = x^2 - 6x + y^3 \) that are parallel to the plane \( 4x - 12y + z = 7 \).

(b) The hyperplane tangent to the 4-dimensional paraboloid \( x_5 = 10 - (x_1^2 + 3x_2^2 + 2x_3^2 + x_4^2) \) at the point \( (2, -1, 1, 3, -8) \).

Question 4. Let \( g(x, y) = \sqrt{xy} \), do the following:

(a) Determine if \( g \) is continuous at \((0, 0)\).

(b) Calculate the partials of \( g \), when \( xy \neq 0 \).

(c) Show that \( g_x(0, 0) \) and \( g_y(0, 0) \) exist by finding the limits.

(d) Determine if the partials are continuous at \((0, 0)\).

(e) Determine if the graph of \( g \) has a tangent plane at \((0, 0)\). It is not necessary, but if you want, show the graph of \( g \).

(f) Determine if \( g \) is differentiable at \((0, 0)\).

Question 5. Do the following:

(a) Verify the Sum Rule for derivatives when \( f(x, y, z) = (xyz^2, xe^{-y}, y\sin xz) \) and \( g(x, y, z) = (x - y, x^2 + y^2 + z^2, \ln(xz + 2)) \).

(b) Verify the Product and Quotient Rules for derivatives when \( f(x, y) = x^2 y + y^3 \) and \( g(x, y) = \frac{x}{y} \).
(c) Verify that the Product Rule holds for the derivative of a cross product in \(\mathbb{R}^3\). That is, directly calculate \(Dh(x)\), for \(h : \mathbb{R}^3 \to \mathbb{R}^3\), \(h(x) = f(x) \times g(x)\), where \(f : \mathbb{R} \to \mathbb{R}^3\) and \(g : \mathbb{R} \to \mathbb{R}^3\) are \(C^1\) functions, and show that it equals the Product Rule \(Dh(x) = Df(x) \times g(x) + f(x) \times Dg(x)\).

**Question 6.** For the function \(F(x, y, z) = 2x^3y + xz^2 + y^3z^5 - 7xyz\), do the following:

(a) Calculate \(F_{xx}, F_{yy}, F_{zz}\).

(b) Calculate all mixed second-partials and verify in all cases that the order in taking partials does not matter.

(c) Determine if \(F_{xy} = F_{yx}\). If so, use the mixed partials theorem to show that this must be the case.

(d) Do the same for \(F_{xz}\) and \(F_{yz}\).

**Question 7.** Let \(f(x, y, z) = \ln\left(\frac{xy}{z}\right)\). Give general formulas for \(\frac{\partial^n f}{\partial x^n}, \frac{\partial^n f}{\partial y^n}\), and \(\frac{\partial^n f}{\partial z^n}\) where \(n \geq 1\). What can you say about the mixed partials?

**Question 8.** Let

\[
F(x, y) = \begin{cases} 
xy \left(\frac{x^2 - y^2}{x^2 + y^2}\right) & \text{if } (x, y) \neq (0, 0) \\
0 & \text{if } (x, y) = (0, 0).
\end{cases}
\]

(a) Find \(f_x(x, y)\) and \(f_y(x, y)\) for \((x, y) \neq (0, 0)\).

(b) Either by hand (using limits), or by using part (a), find the partial derivatives \(f_x(0, y)\) and \(f_y(x, 0)\).

(c) Find the values to \(f_{xy}(0, 0)\) and \(f_{yx}(0, 0)\). Reconcile this with Theorem 4.3 in the text on page 137, on the equality of mixed partials.

**Question 9.** Do the following:

(a) If \(f(x, y) = \sin(xy)\) and \(x = s + t\) and \(y = s^2 + t^2\), find \(\frac{\partial f}{\partial s}\) and \(\frac{\partial f}{\partial t}\), both by direct substitution, and by means of the Chain Rule.

(b) Suppose that \(z = x^2 + y^3\), where \(x = uv\) and \(y\) is also some function of \(u\) and \(v\). Suppose further that, when \((u, v) = (2, 1)\), then \(\frac{\partial y}{\partial v} = 0\). Determine \(\frac{\partial z}{\partial v}(2, 1)\).