Question 1. Do the following:

(a) For the $C^1$ function $z = f(x, y)$, let $x = 2uv$ and $y = u^2 + v^2$. Show that
\[
\frac{\partial z}{\partial u} \frac{\partial z}{\partial v} = 2x \left[ \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 \right] + 4y \frac{\partial z}{\partial x} \frac{\partial z}{\partial y}.
\]

(b) For the $C^1$ function $w = g(u)$, where $u = \frac{x^2-y^2}{x^2+y^2}$, show
\[
x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} = 0.
\]

Question 2. For \( f(x) = (x^2, \cos 3x, \ln x) \), and \( g(s, t, u) = s + t^2 + u^3 \), calculate \( D(f \circ g) \) in two ways: (1) by first writing an expression for \( (f \circ g) \) and then calculating the derivative, and (2) by using the Chain Rule with the derivatives \( Df \) and \( Dg \).

Question 3. Using Example 6 on pages 152-3 of the text, determine the operators \( \frac{\partial^2}{\partial x^2} \) and \( \frac{\partial^2}{\partial y^2} \) in terms of the polar coordinate partial derivative operators \( \frac{\partial^2}{\partial r^2} \), \( \frac{\partial^2}{\partial \theta^2} \), \( \frac{\partial}{\partial r} \), \( \frac{\partial}{\partial \theta} \), and \( \frac{\partial}{\partial \theta} \). (Hint: You will need the Product Rule for differentiation here.)

Question 4. The equation
\[
F(x, y) = 0,
\]
defines \( y \) implicitly as a function of \( x \), so we can write \( y(x) \). Do the following:

(a) If both \( F \) and \( y(x) \) are assumed to be \( C^1 \), show that
\[
\frac{dy}{dx} = -\frac{F_y(x, y)}{F_x(x, y)}.
\]
provided that $F_y(x,y) \neq 0$.

(b) The 2-level set of $F(x,y) = \sin(xy) - x^2y^7 + 2e^{2y^2-2}$ is shown in the graph in Figure 1. Use part (a) to find the equation of the line tangent to this curve at the point $(0,1)$.

**Question 5.** The equation

$$G(x, y, z) = 0,$$

defines $z$ implicitly as a function of $x$ and $y$, so we can write $z(x, y)$. Do the following:

(a) If both $G$ and $z(x,y)$ are assumed to be $C^1$, show that

$$\frac{\partial z}{\partial x} = -\frac{F_x(x, y, z)}{F_z(x, y, z)}, \quad \text{and} \quad \frac{\partial z}{\partial y} = -\frac{F_y(x, y, z)}{F_z(x, y, z)}$$

provided that $F_z(x, y, 0) \neq 0$.

(b) Calculate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, where $z$ is given implicitly by the equation

$$x^3z + y\cos z + \frac{\sin y}{z} = 0.$$

**Question 6.** Suppose $f(x,y)$ is a differentiable function of three variables.

(a) Explain what the quantity $\nabla f(x,y,z) \cdot (-k)$ represents.

(b) How does $\nabla f(x,y,z) \cdot (-k)$ relate to $\frac{\partial f}{\partial z}$?

**Question 7.** Calculate the directional derivative of $f(x, y, z) = xe^{y^3} + 1$ at the point $a = (2, -1, 0)$ in the direction of $v = i - 2j + 3k$.

**Question 8.** Calculate the plane tangent to the surface whose equation is $x^2 - 2y^2 + 5xz = 7$ at the point $(-1, 0, -\frac{6}{5})$ in two ways:

(a) By solving for $z$ in terms of $x$ and $y$ and using the partial derivative of $z$, and

(b) By considering this surface the level set of a function and using the function’s gradient.

**Question 9.** Do the following for the surface defined by $x^3z + x^2y^2 + \sin(yz) = -3$:

(a) Find an equation for the tangent plane to the surface at the point $(-1,0,3)$.

(b) Find a set of parametric equations defining the line *normal* to the surface at the point $(-1,0,3)$. (The line normal to a surface at a point is the line perpendicular to the tangent space of the surface at that point that passes through the point.

(c) Find a general formula for the normal line to a surface defined by $F(x, y, z) = 0$ at the point $(x_0, y_0, z_0)$. 
Question 10. Let $F(x, y) = c$ define a curve $S_c$ in the plane. Suppose $(x_0, y_0)$ is a point of $S_c$ such that $\nabla F(x_0, y_0) \neq 0$. Show that near $(x_0, y_0)$ the curve $S_c$ can either be represented as the graph of a function $y = f(x)$ or of a function $x = g(y)$.

Question 11. Let $S$ be the points defined by the equation $\sin xy + e^{xz} + x^3y = 1$. Near which points can we describe $S$ as the graph of a $C^1$ function $z = f(x, y)$? Can you find an expression for $f(x, y)$ in this case? Now describe the set of points $(x_0, y_0, z_0) \in S$ where you cannot describe $S$ as the graph of a function $z = f(x, y)$.