Question 1. Can you solve
\[ x_2y_2 - x_1 \sin y_1 = 5 \]
\[ x_2 \sin y_1 + x_1 y_2 = 2 \]
for \( y_1, y_2 \) as functions of \( x_1, x_2 \) near the point \((x_1, x_2, y_1, y_2) = (2, 3, \pi, 1)\)? What about near the point \((x_1, x_2, y_1, y_2) = (0, 2, \frac{\pi}{2}, \frac{5}{2})\)?

Question 2. Consider the system:
\[
\begin{cases}
  x_1 y_2^2 - 2x_2 y_3 = 1 \\
  x_1 y_1^2 + x_2 y_2 - 4y_2 y_3 = -9 \\
  x_2 y_1 + 3x_1 y_2^2 = 12
\end{cases}
\]

(a) Show that, near the point \((x_1, x_2, y_1, y_2, y_3) = (1, 0, -1, 1, 2)\), it is possible to solve for \(y_1, y_2,\) and \(y_3\) in terms of \(x_1\) and \(x_2\).

(b) From the results of part (a), we may consider \(y_1, y_2,\) and \(y_3\) to be functions of \(x_1\) and \(x_2\). Use implicit differentiation and the Chain Rule to evaluate each of \(\frac{\partial y_1}{\partial x_1}(1, 0), \frac{\partial y_2}{\partial x_1}(1, 0),\) and \(\frac{\partial y_3}{\partial x_1}(1, 0)\).

Question 3. Let \( x(t) \in \mathbb{R}^n \) be a differentiable curve such that \( \| x(t) \| = c > 0 \), for all \( t \). Show that \( x(t) \cdot x'(t) = 0 \), for every \( t \).

Question 4. For the curve \( x(t) = (t \sin t, t \cos t, t^2) \), do the following:

(a) Calculate the speed, velocity and acceleration of the curve.

(b) Show that the curve, for \( t \in [-20, 20] \), lies on the paraboloid \( S \) defined by the equation \( z = x^2 + y^2 \).

Question 5. Do the following:

(a) Sketch the path of \( x(t) = (t, t^3 - 2t + 1) \).

(b) Calculate the line tangent to \( x \) at \( t = 2 \).

(c) Eliminate the parameter \( t \) by writing the curve as a function \( y = f(x) \).

(d) Verify by calculation that the line you calculated in part (b) is the same line one calculates as the tangent line to the graph of \( f \) in part (c).
Question 6. Verify the Product Rule for calculating the velocity of a curve in $\mathbb{R}^3$, given as the cross product of two other curves in $\mathbb{R}^3$. That is, for two differentiable curves $x, y : \mathbb{R} \to \mathbb{R}^3$, define $z = x \times y$ and show that

$$\frac{dz}{dt} = \frac{dx}{dt} \times y + x \times \frac{dy}{dt}.$$  

Question 7. Calculate the length of each of the paths:

(a) $x(t) = 7i + tj + t^2k$, for $1 \leq t \leq 3$.

(b) $y(s) = \left( \ln s, \frac{s^2}{2}, \sqrt{2}s \right)$, with $s \in [1, 4]$.

Question 8. For the path $x(t) = (e^{-t} \cos t, e^{-t} \sin t)$, do the following:

(a) Argue that the path spiral in toward the origin at $t \to \infty$.

(b) Show that, for any choice of $a \in \mathbb{R}$, the improper integral

$$\int_a^\infty ||x'(t)|| \; dt$$

converges.

(c) Interpret what the result from part (b) says about the path $x$.

Question 9. Sketch the following vector fields:

(a) $F = (x, x^2)$.

(b) $G = (y, -x, 2)$.

Question 10. Show that the vector field $F = 2xi + 2yj - 3k$ is a gradient vector field. Describe the equipotential surfaces of $F$ both in words and in sketches.

Question 11. Show that the curve $x(t) = (\sin t, \cos t, e^{2t})$ is a flow line off the vector field $F = (y, -x, 2z)$.  