Motives over Symmetric Monoidal Categories

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Our purpose is to define a category \( \tilde{\text{Mot}}_C \) of motives over a symmetric monoidal category \((C, \otimes, 1)\). We introduce a notion of correspondences between (not necessarily commutative) monoid objects in \( C \) and consider the category \( ALG_C \) obtained by enlarging the category \( \text{Mon}(C) \) of monoid objects in \( C \) with correspondences. Then, a motive over \( C \) is a triple \((R, e, m)\) where \( R \) lies in \( \text{Mon}(C) \), \( e \) is a projector from \( R \) to itself in the category \( ALG_C \) and \( m \in \mathbb{Z} \). The morphism sets in \( \tilde{\text{Mot}}_C \) are defined in terms of bivariant cyclic cohomology. In this context, we show that the Connes periodicity operator \( S \) defines a morphism

\[
S : \tilde{\text{Mot}}_C((R, e, m), (R', e', m')) \longrightarrow \tilde{\text{Mot}}_C((R, e, m) \otimes \mathbb{L}^{\otimes 2}, (R', e', m'))
\]

(0.1)

where \( \mathbb{L} := (1, 1, -1) \) denotes the Lefschetz motive in \( \tilde{\text{Mot}}_C \). Hence, the tower of periodicity operators defining bivariant periodic cyclic cohomology can be interpreted in terms of successive twists by the Lefschetz motive.