Final exam — 110.109 Calculus II — December 12, 2008

Instructions: You must show all of your work clearly and legibly to receive full credit. No calculators, books, or notes are allowed for this exam. Please PRINT your name and section number.

NAME: SECTION:

I will not cheat.
Signature:

Please do not write on this table!

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
TOTAL	200	

1. Consider the parametric curve $C : \{x = t^2, y = t^3 - 3t\}$.

$a)$: Verify that the curve passes through the point $(3, 0)$. For what values of t?

$b)$: Determine the equation of the line(s) tangent to the curve at the point $(3, 0)$.

2. $a)$: Find a polar equation for the curve represented by the Cartesian equation $x + y = 0$.

$b)$: Find the points on the polar curve $r = 3\cos\theta$ where the tangent line is horizontal or vertical.

3. Determine whether the following sequences have a limit. If the limit exists, find it. Explain!

$a) : a_n = \sqrt{n+1} - \sqrt{n}.$

$b) : a_n = \frac{n!}{3^n}.$

4. Consider the improper integral $\int_0^1 \frac{1}{x^p} dx$.

$a)$: Show that the integral diverges if $p \geq 1$.

$b)$: Show that the integral converges if $p < 1$, and determine its value.

5. $a)$: Determine whether the improper integral $\int_0^\infty \frac{x}{1+xe^x}dx$ converges or diverges.

$b)$: Show that if the series $\sum a_n$ is absolutely convergent, then the series $\sum \frac{(n+1)a_n}{n}$ is absolutely convergent.

6. Find the radius of convergence and interval of convergence for the power series

$a) : \sum_{n=1}^{\infty} \frac{x^n}{n3^n}$.

$b) : \sum_{n=1}^{\infty} n!(x+1)^n$.

7. Find the sum of the series

$a) : \sum_{n=0}^{\infty}(n+1)x^n \ (|x| < 1).$

$b) : \sum_{n=1}^{\infty} \frac{n}{2^n}.$

8. Use the geometric series to find the power series representation for the function (*all coefficients of the series must be determined*). Determine the *interval* of convergence:

$a) : f(x) = \frac{1}{x^2+4}$.

$b) : f(x) = \ln(1 + x)$.

9. Find the Taylor series for the function at the given value (*all coefficients of the series must be determined*):

$a) : f(x) = e^x, a = -1.$

$b) : f(x) = x^4, a = 1.$

10. a) Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ correct up to two decimal places.

b) Show that $|\cos x - 1 + \frac{x^2}{2}| \leq \frac{x^3}{6}$ for all $x > 0$.