

<i>problems</i>	1 – 9	10 – 15	16 – 17	<i>total</i>
<i>scores</i>				

Final Exam, December 8, Calculus II (109), Fall, 2010, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name: _____ Date: _____

NO CALCULATORS, NO PAPERS, SHOW WORK. All of the problems are 2 point problems. 1 point will be given where it seems appropriate. You are required to show your work. If you have the correct answer but have no work, then you get no credit. If you have a lot of good work, but don't have the correct answer, that doesn't work so well. The answers that will be graded are those you put on the answer sheet at the beginning of the exam. (50 points total)

TA Name and section: _____

Problems are in order they were taught, not in order of difficulty or time needed.

MAKE SURE YOU PUT YOUR ANSWERS IN THE BOXES IN THE FIRST 3 PAGES.

Problem # 1 answer:

Problem # 2 answer:

Problem # 3 answer:

Problem # 4 answer:

Problem # 5 answer:

Problem # 6 answer:

Problem # 7 answer:

Problem # 8 answer:

Problem # 9 answer:

Problem # 10 answer:

Problem # 11 answer:

Problem # 12 answer:

Problem # 13 answer:

Problem # 14 answer:

(a)

(b)

Problem # 15 answer:

(a)

(b)

(c)

Problem # 16 answer:

(a)

(b)

(c)

Problem # 17 answer:

(a)

(b)

(c)

(d)

1. (2 points) $(x^3 + 1)$ is divisible by $x + 1$. Write $\frac{3}{(x^3+1)}$ in terms of partial fractions.

6

2. (2 points) Compute $\int \frac{1}{x^2-x+1} dx$.

3. (2 points) Compute $\int_0^1 \frac{3}{x^3+1} dx$.

4. (2 points) Solve $yy' = x^3 + y^2x^3$ if $y = 2$ when $x = 0$. Extra scrap page follows

Scrap paper.

5. (2 points) Solve $y' + 3x^2y = e^{-x^3}$ if $y = 1$ when $x = 0$. Extra scrap page follows.

Scrap paper.

6. (2 points) Find the area trapped by the curve $x = t^3 - 4t$ and $y = 4 - t^2$ where $y > 3$ and $x > 0$.

7. (2 points) Let $r = \sin(\theta) + \cos(\theta)$. Find the xy-coordinates of the point on the curve maximum distance from the origin.

8. (2 points) Let $r = \sin(\theta) + \cos(\theta)$. For what values of θ (between $-\pi$ and π) does the curve go through the origin?

9. (2 points) Let $r = \sin(\theta) + \cos(\theta)$. Find the slope of the tangent line to this curve at $\pi/2$.

10. (2 points) Let $r = \sin(\theta) + \cos(\theta)$. Find the area inclosed by this curve.

11. (2 points) Let $r = \sin(\theta) + \cos(\theta)$. Find the length of this curve. (This gets ugly, but if you persist, it simplifies more or less completely.) Extra scrap page follows.

Scrap paper.

12. (2 points) Find the first 2 non-zero terms for the Taylor series for $\tan^{-1}(x)$ around $a = 1$.

13. (2 points) Find the first 2 non-zero terms for the Taylor series for $\sin^{-1}(x)$ around $a = 0$.

14. (4 points) (a) Find the quadratic approximation (i.e. the second Taylor polynomial) for $x^3 - 1$ around $a = 0$. (b) Evaluate at $x = .1$.

15. (6 points) (a) Find the quadratic approximation (i.e. the second Taylor polynomial) for $x^3 - 1$ around $a = 1$. (b) Evaluate at $x = 1.1$. (c) What is the difference between the actual value of $x^3 - 1$ at $x = 1.1$ and this approximation?

16. (6 points)

- (a) Find the first 2 non-zero terms for the Taylor series for $f(x) = \int_0^{x^2} \frac{dz}{1+z}$ around $a = 0$.
- (b) Use this to approximate $f(.2)$.
- (c) For how many decimal places is this approximation correct?

Extra scrap page follows.

Scrap paper.

17. (8 points) Let $f(x) = 2^x$. It is a fact that $2^{2.1} = 4.2870938\dots$. Use the approximations $\ln(2) = .69315$, $(\ln(2))^2 = .48045$, and $(\ln(2))^3 = .33302$. Keep 5 decimal places in all computations.

(a) Find, in closed form, i.e. without using the above approximations, the first 3 non-zero terms of the Taylor series for 2^x around $a = 2$ (i.e. the quadratic approximation in $(x - 2)$, or, phrased yet another way, the second Taylor polynomial.

(b) Using these three terms and the above approximations, approximate $2^{2.1}$.

(c) Give the explicit formula in this case for the remainder term.

(d) Assuming you know that $2^{2.1} < 6$ (this makes the numbers work out easily), use the formula for the remainder to bound the remainder.

(extra page follows)

